

Iterative Methods to Solve Otherwise-Intractable Grid Optimization Models

Alexandra Newman

PSERC, 23 September 2026

Colorado School of Mines



COLORADO SCHOOL OF MINES

An Optimization Model

$$\begin{array}{ll}\min & cx \\ \text{subject to} & Ax = b \\ & x \geq 0 \text{ and integer}\end{array}$$

Risk Aware Optimal Power Flow

Remi Akinwonmi Alexandra Newman Daniel Bienstock

PSERC, 23 September 2026

Colorado School of Mines

Columbia University



COLORADO SCHOOL OF MINES

Risk Aware Optimal Power Flow

Flow Concentration and Multi-Hazard Grid Vulnerability

Cost-optimal grid dispatch concentrates power flow as an endogenous result of Optimal Power Flow (OPF). Power systems infrastructure is increasingly vulnerable to high-impact low-frequency (HILF) disruptive events.

Correlated Hazard Drivers

- **Fire Weather:** Thermally driven line failures, where survival probability declines as power flow increases
- **Cyber-Physical Attacks:** Targeted, adversarial removal of vulnerable substation and system infrastructure
- **Geomagnetic Disturbances:** Spatially correlated component losses due to storm footprints

Objective

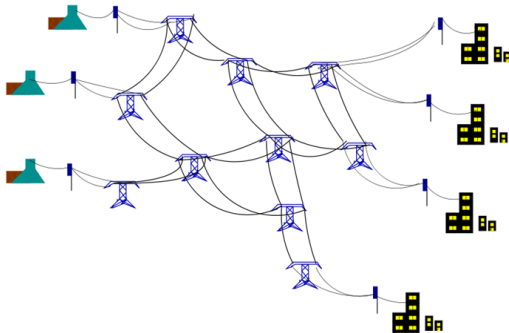
Develop a robust OPF model using an optimization framework that facilitates feasible power flow solutions under extreme contingencies. This is designed to **reduce a flow functional** (i.e. reduce exposure to risk) while achieving minimal increase in operational cost.

Optimal Power Flow (OPF) Formulation

Objective: Minimize total generation cost while balancing supply and demand.

Constraints:

- Power balance at each bus
- Line flow limits and thermal ratings
- Generator operating limits and ramp rates
- Voltage magnitude constraints



Power Grid Network

Single-Period DC Optimal Power Flow (DCOPF)

1. Dispatch (objective)

Minimize total generation cost over all units:

$$\min_{P_g, \theta, P} \sum_{i \in \mathcal{G}} \sum_{g \in \mathcal{G}_i} \left(C_g^0 + C_g^1 P_g + C_g^2 P_g^2 \right)$$

Operating limits

$$P_g^{\min} \leq P_g \leq P_g^{\max} \quad \forall g \in \mathcal{G}_i, i \in \mathcal{G}$$

$$-S_{ij}^{\max} \leq P_{ij} \leq S_{ij}^{\max} \quad \forall (i, j) \in \mathcal{E}$$

- P_g : active power of unit g
- $S_{ij}^{\max}$: thermal rating of branch (i, j)
- C_g^0, C_g^1, C_g^2 : cost coefficients

2. Network

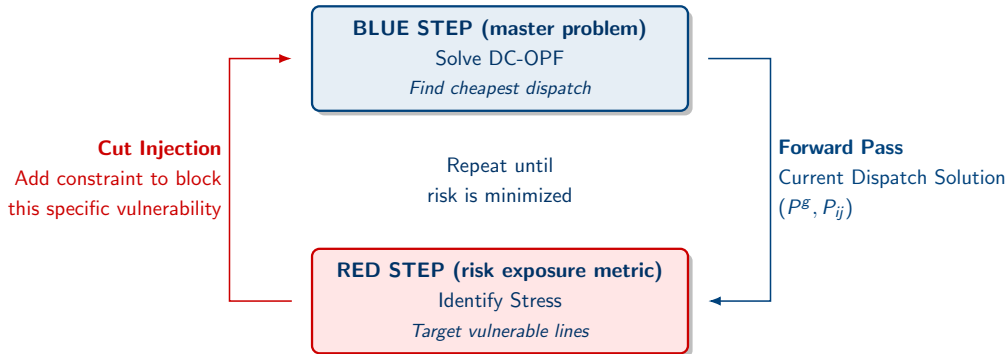
Branch level — flow is proportional to the angle difference:

$$P_{ij} = B_{ij} \theta_{ij}, \quad \theta_{ij} = \theta_i - \theta_j \quad \forall (i, j) \in \mathcal{E}$$

Bus level — net injection equals net outflow:

$$\sum_{g \in \mathcal{G}_i} P_g - P_{di} = \sum_{(i, j) \in \mathcal{E}} P_{ij} - \sum_{(j, i) \in \mathcal{E}} P_{ji} \quad \forall i \in \mathcal{N}$$

P_{di} is the demand at bus i .



Risk-Aware Master Problem and Endogenous Risk Metrics

1. Robust DC-OPF (master problem)

Blue minimizes generation cost plus a penalty on a risk variable R :

$$\min_{P^g, \theta, P} \sum_{k \in \mathcal{G}} F_k(P_k^g) + \lambda R$$

subject to the DC-OPF constraints plus accumulated cuts $R \geq$ (subgradient of risk).

- R : worst-case risk variable
- λ : risk-weight parameter
- $F_k(\cdot)$: generation cost

2. Red subproblem (risk metrics)

Line level — penalize the worst single branch over the set $\mathcal{S}$:

$$R \geq |P_{km}| \quad \text{or} \quad R \geq r_{km} P_{km}^2 \quad \forall (k, m) \in \mathcal{E}$$

(max active flow; max Joule loss).

Bus level — aggregate flow incident to bus i :

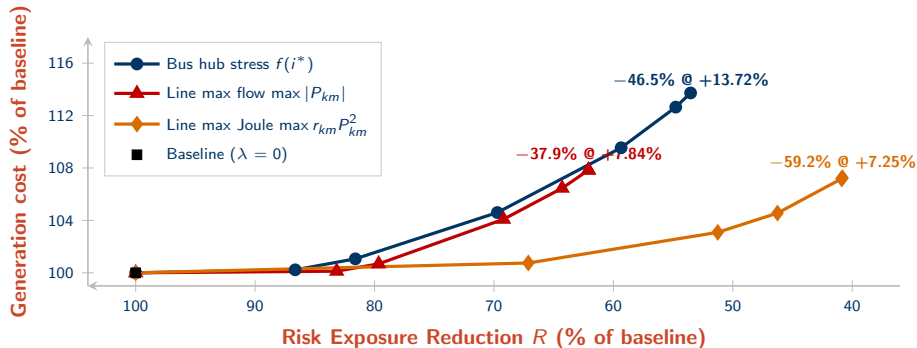
$$f(i) = \sum_{(k,m) \in \delta(i)} |P_{km}|$$

Red selects the worst hub i^* ; Blue adds *one* aggregate cut:

$$R \geq \sum_{(k,m) \in \delta(i^*)} \text{sign}(P_{km}^{(n)}) P_{km}$$

Results: Cost–Risk Frontier

DC-OPF on **pglib GOC 30,000-bus**, Gurobi 13.0.0. Baseline ($\lambda = 0$) cost $C_0 = 1,714,285$ (= 100%).



Conclusion

- Risk-aware DC-OPF cutting-plane; every frontier point run converges in < 90 s on 30,000 buses
- Risk metric extended from branch worst case to *bus stress*

Next Steps

- Random walk counterfactual analysis
- **GEACRA cyber-risk integration** (with TAMU)

Spatial Decomposition of a Large-Scale Linear Program for Power Sector Capacity Planning



Spatial Decomposition of a Large-Scale Linear Program for Power Sector Capacity Planning

Alex Derenchuk Alexander Zolan Alexandra Newman Maxwell Brown

PSERC, 23 September 2026

Colorado School of Mines

National Laboratory of the Rockies



COLORADO SCHOOL OF MINES

Regional Energy Deployment System (ReEDS)

- ReEDS is a capacity expansion model for the power sector that simulates continent-scale evolution of the bulk power system decades into the future

Inputs

- Existing energy infrastructure
- Fuel prices and supply curves
- Time-varying demand and generation
- Cost and availability of new infrastructure
- Policies (e.g., emissions limits, tax credits)



Outputs

- Power plant builds/retires
- Total system cost and electricity prices
- Hourly operation of the power grid for representative intervals



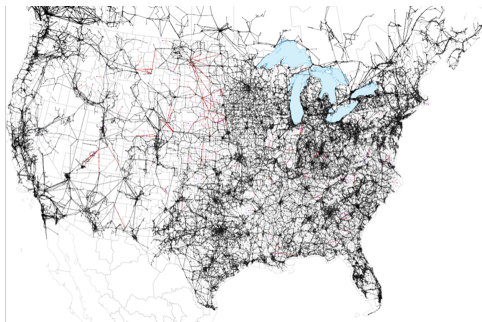
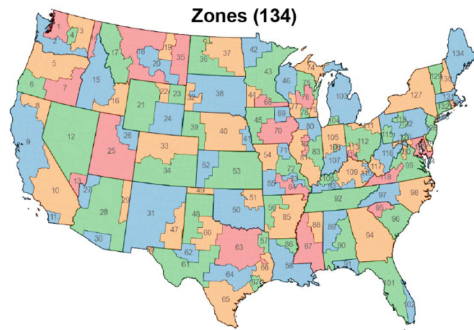
Model Tractability Requires Reduced Temporal Resolution



Selection of representative days for typical ReEDS run



Model Tractability Requires Reduced Spatial Resolution



Can We Solve High-resolution ReEDS Models?

Motivation

- Currently, ReEDS models with high-temporal and/or high-spatial-fidelity require several hours to run, or are entirely intractable, which hinders analysis.

Research Questions

- Can we apply spatial decomposition to make difficult ReEDS instances tractable?
- How do performance improvements vary across high spatial fidelity and high temporal fidelity models?
- Can arbitrary cross-region policy constraints and flow variables be effectively decomposed?

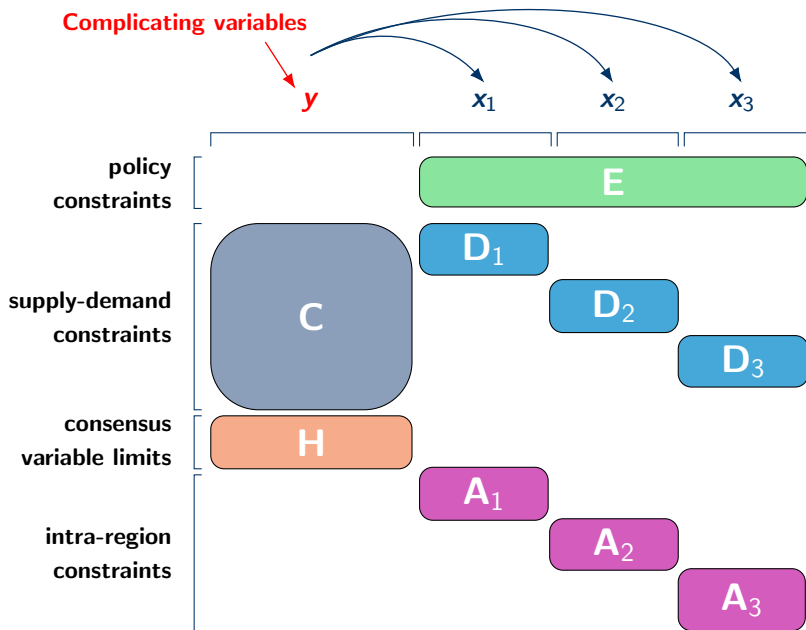


Capacity Expansion Model Formulation

$$\begin{aligned} \min \quad & \sum_{g \in \mathcal{G}} \mathbf{c}_g^T \mathbf{x}_g + \mathbf{f}^T \mathbf{y} && \text{(Operations and Investment costs)} \\ \text{s.t.} \quad & \sum_{g \in \mathcal{G}} E_g \mathbf{x}_g \geq \mathbf{e} && \text{(policy constraints)} \\ & C_g \mathbf{y} + D_g \mathbf{x}_g = \mathbf{d}_g \quad \forall g \in \mathcal{G} && \text{(supply-demand constraints)} \\ & H \mathbf{y} \leq \mathbf{h} && \text{(consensus variable limits)} \\ & A_g \mathbf{x}_g \leq \mathbf{b}_g \quad \forall g \in \mathcal{G} && \text{(intra-region constraints)} \end{aligned}$$



Complicating variables



Monolith vs. Consensus Formulation

$$\min \sum_{g \in \mathcal{G}} \mathbf{c}_g^T \mathbf{x}_g + \mathbf{f}^T \mathbf{y}$$

s.t.

$$\sum_{g \in \mathcal{G}} E_g \mathbf{x}_g \geq \mathbf{e}$$

$$C_g \mathbf{y} + D_g \mathbf{x}_g = \mathbf{d}_g \quad \forall g \in \mathcal{G}$$

$$H \mathbf{y} \leq \mathbf{h}$$

$$A_g \mathbf{x}_g \leq \mathbf{b}_g \quad \forall g \in \mathcal{G}$$

$$\min \sum_{g \in \mathcal{G}} \mathbf{c}_g^T \mathbf{x}_g + \sum_{g \in \mathcal{G}} \mathbf{f}^T \mathbf{y}_g$$

$$\text{s.t. } \mathbf{y}_g = \mathbf{y} \quad \forall g \in \mathcal{G}$$

$$\sum_{g \in \mathcal{G}} \mathbf{z}_g \geq \mathbf{e}$$

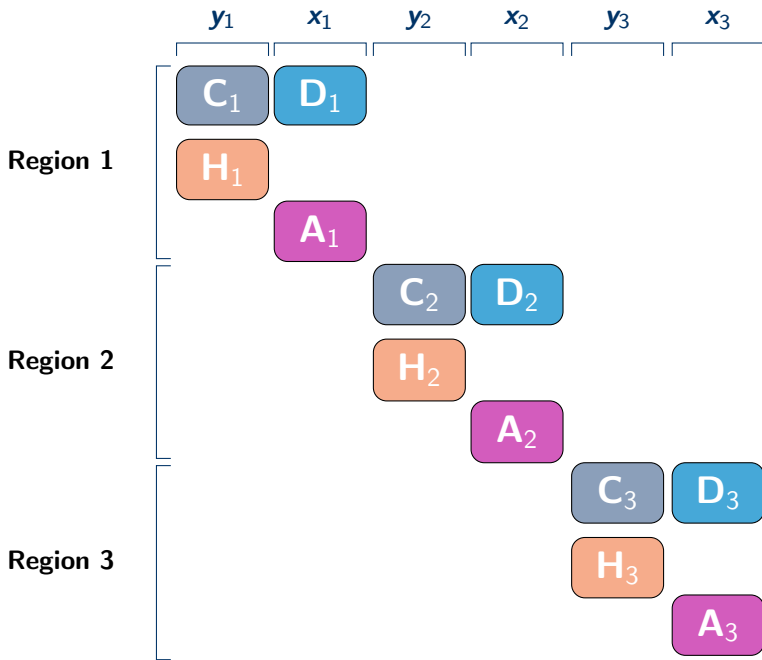
$$\mathbf{z}_g = E_g \mathbf{x}_g \quad \forall g \in \mathcal{G}$$

$$C_g \mathbf{y}_g + D_g \mathbf{x}_g = \mathbf{d}_g \quad \forall g \in \mathcal{G}$$

$$H_g \mathbf{y}_g \leq \mathbf{h}_g \quad \forall g \in \mathcal{G}$$

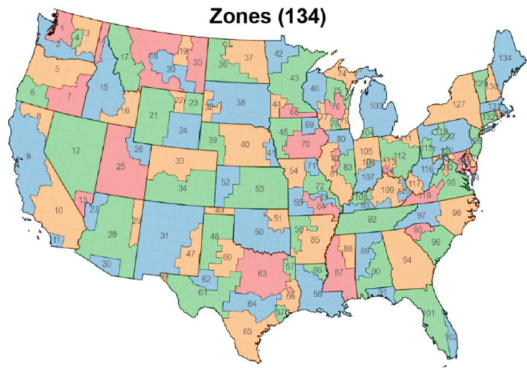
$$A_g \mathbf{x}_g \leq \mathbf{b}_g \quad \forall g \in \mathcal{G}$$





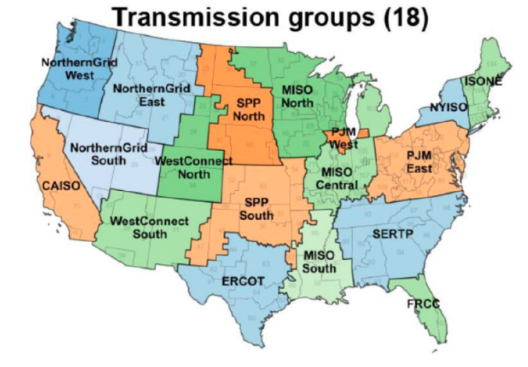
Case Study

- Testbed: standard ReEDS capacity expansion model solve: 134 regions across U.S., 33 representative days.



Case Study

- For spatial decomposition, split U.S. into 18 transmission group regions. Only complicating variables were power flow between regions – no policy constraints



Monolith

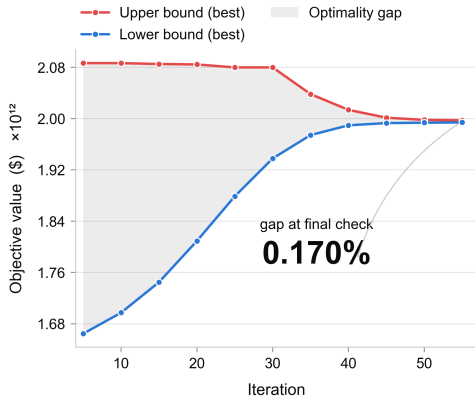
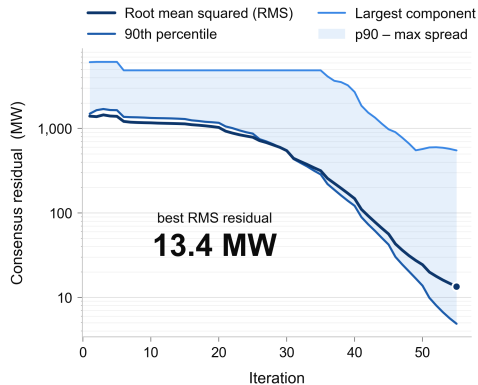
- Monolith LP had 1.27M rows, 1.60M columns, 6.29M nonzeros (988 dense columns) after presolve.
- Solve time: **573 seconds** with barrier+crossover on 8 threads

Spatial Decomposition

- Divided U.S. model into 18 regions, solved to 0.2% optimality gap
- Required 55 iterations at an average of 34 seconds per iteration
- Total time: **3031 seconds**



Case Study Results



Conclusion and Further Work

- A spatial decomposition methodology using was developed for a large-scale linear capacity expansion model.
- Tests on flow decomposition show convergence to a tight objective function bound fairly quickly.
- Further work will include testing policy constraint decomposition, higher-fidelity models, and intertemporal solves.

