IBR Stability Analysis Methods: Pitfalls and Misconceptions

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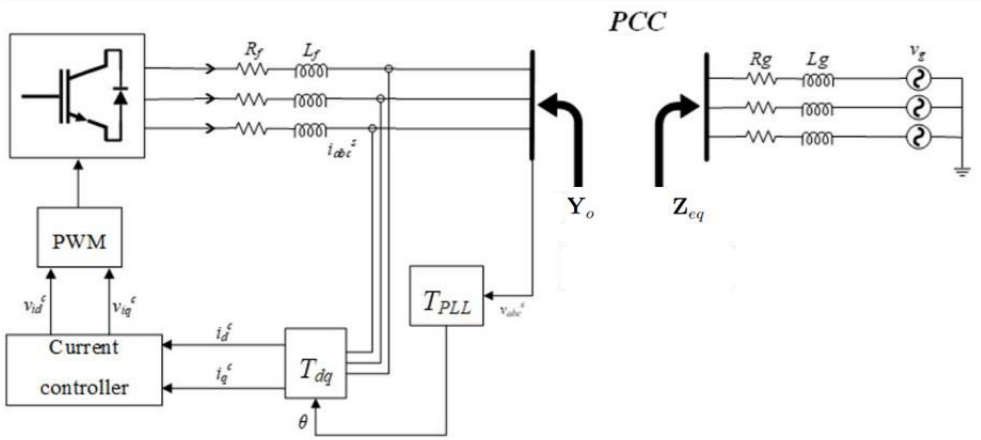
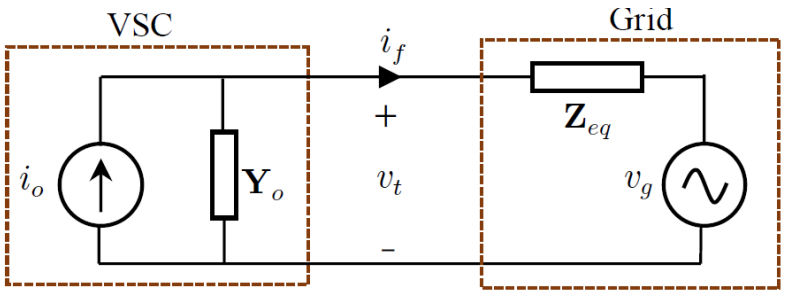
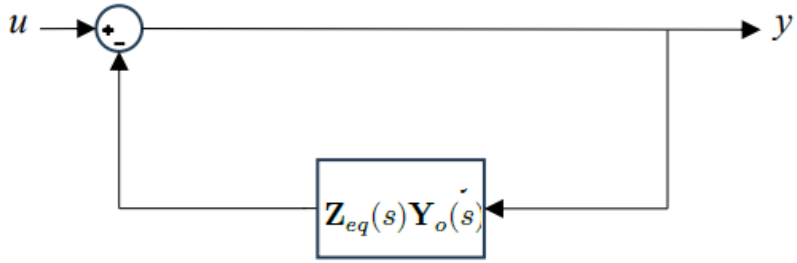
Outline

- Brief overview of frequency-domain stability analysis (e.g., impedance-based stability method)
- Equivalence between state-space and transfer-matrix representations
- Stability and robustness assessment in SISO systems
- Stability and robustness analysis of MIMO systems in the context of inverter-based resources (IBRs)
- Summary
- Acknowledgement of the contributors

Impedance based stability analysis





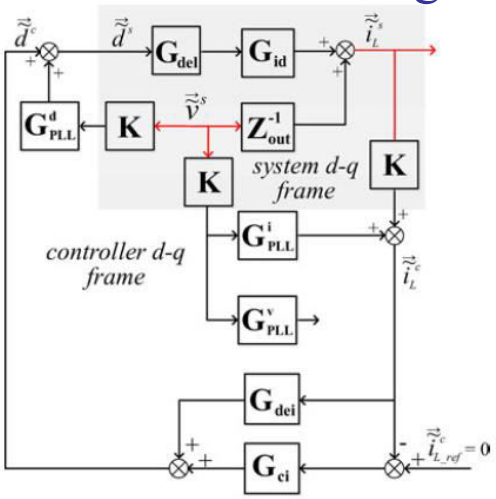


$$i_f(s) = \left(i_o(s) - \mathbf{Y}_o(s)v_g(s)\right) \times (\mathbf{I} + \mathbf{Z}_{eq}(s)\mathbf{Y}_o(s))^{-1}$$

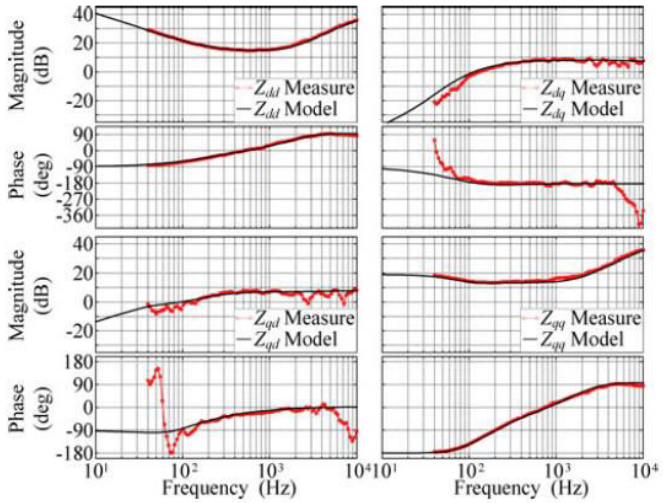
$$\mathbf{G}(s) = \mathbf{Z}_{eq}(s)\mathbf{Y}_o(s)$$

IBRs modeling

White box modeling



Black box modeling



$$Y(s) = \begin{bmatrix} Y_{11}(s) & Y_{12}(s) \\ Y_{21}(s) & Y_{22}(s) \end{bmatrix}$$

$$Z(s) = \begin{bmatrix} Z_{11}(s) & Z_{12}(s) \\ Z_{21}(s) & Z_{22}(s) \end{bmatrix}$$

- For MIMO black-box models, the frequency-domain admittance or impedance matrix, Y(w) or Z(w), may be provided directly from measurements. The corresponding S-domain transfer matrix, Y(s) or Z(s), can then be readily derived by applying matrix fitting.

B. Wen, D. Boroyevich, R. Burgos, P. Mattavelli, and Z. Shen, "Analysis of D–Q small-signal impedance of grid-tied inverters," IEEE Transactions on Power Electronics, vol. 31, no. 1, pp. 675–687, Jan. 2016.

MIMO model

IEEE Transactions on Power Apparatus and Systems, Vol. PAS-95, no. 4, July/August 1976

SUBSYNCHRONOUS OSCILLATIONS
PART 1 — COMPREHENSIVE SYSTEM STABILITY ANALYSIS

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Power Technologies, Incorporated
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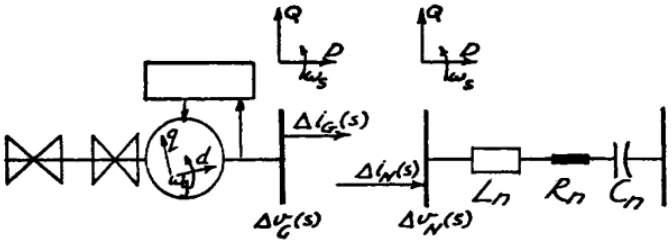


Figure 2 Separation Of System For Observation At Generator Terminals

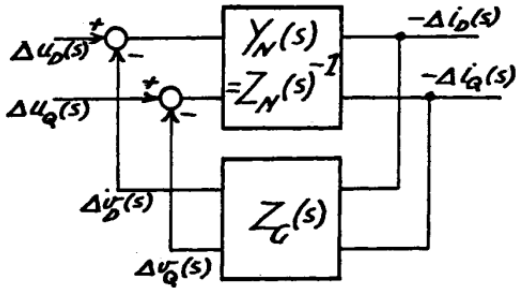


Figure 3 Feedback Relationship Between Generating Unit And System

Coupling Between Shafts in Separate Plants

To illustrate the possibility of coupling between the shafts of generators in different plants, consider the case where generators 1 and 2 are mechanically identical, though separated by a 100-mile series compensated transmission line. We first make stability analysis representing shaft torsional effects in only one machine at a time. Figure 12 considers the 249.5 r/s torsional mode at machine 1 only, while Figure 14 considers this mode at machine 2 only with the shaft effects of machine 1 being neglected. Both figures show closed Nyquist plots with the observation point at the terminal bus of generator 1; both show a significant "bulge" at the frequency of 249.5 r/s, but both indicate stability.

Now Figure 15 shows the closed loop Nyquist plot with the 249.5 r/s shaft torsional mode represented in machines 1 and 2, and indicates instability.

This shows that there can be significant oscillatory exchange of energy between the shafts of generators separated by significant distance. Whether this effect would be important in any given series compensated system can only be indicated by specific studies. This illustrative example suggests, however, that the possibility should be investigated, and that consideration of shaft dynamics of only one generator at a time could fail to reveal possible modes of instability.

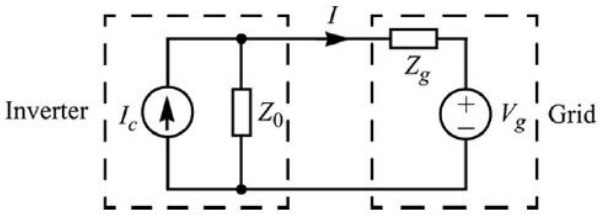
SISO models

IEEE TRANSACTIONS ON POWER ELECTRONICS, VOL. 26, NO. 11, NOVEMBER 2011 3075

Letters

Impedance-Based Stability Criterion for Grid-Connected Inverters

Jian Sun



$$I(s) = \left[I_c(s) - \frac{V_g(s)}{Z_0(s)}\right] \cdot \frac{1}{1 + Z_g(s)/Z_0(s)}.$$

Fig. 3. Small-signal representation of an inverter-grid system.

INPUT FILTER CONSIDERATIONS IN DESIGN AND APPLICATION OF SWITCHING REGULATORS

R. D. MIDDLEBROOK

California Institute of Technology
Pasadena, California

D. Middlebrook, “Input filter considerations in design and application of switching regulators,” in *Proc. IEEE Ind. Appl. Soc. Annu. Meeting*, 1976, pp. 366–382.

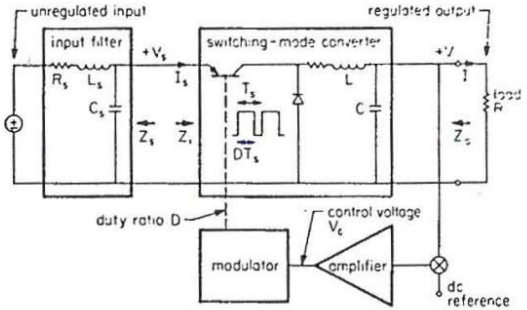


Fig. 1. Essential elements of a switching-mode regulator with input filter. The block diagram is general, and a single-section LC input filter and a buck converter are shown as typical realizations.

State-space and transfer function modeling equivalence

- The state-space and transfer matrix (or transfer function matrix) representations can be used for presenting the dynamic behavior of a system.

- One can move from a state space model

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

To transfer matrix form

$$G(s) = C(sI - A)^{-1}B + D$$

- Girven a transfer matrix G(s)

$$Y(s) = G(s)U(s)$$

A state-space realization (A, B, C, D) can always be constructed such that

$$G(s) = C(sI - A)^{-1}B + D$$

State-space and transfer function modeling equivalence

Example:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

$$A = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix} \quad B = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad C = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad D = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

```
% Define system matrices
A = [0 1; -2 -3];
B = [1 0; 0 1];
C = [1 0; 0 1];
D = [0 0; 0 0];

% Create state-space model
sys_ss = ss(A,B,C,D);

% Convert to transfer function (matrix form)
sys_tf = tf(sys_ss);

% Display results
disp('Transfer matrix G(s) = ');
sys_tf
```

$$\begin{bmatrix}Y_1(s)\\ Y_2(s)\end{bmatrix} = \begin{bmatrix}\frac{s+3}{s^2+3s+2} & \frac{1}{s^2+3s+2}\\ \frac{-2}{s^2+3s+2} & \frac{s}{s^2+3s+2}\end{bmatrix}\begin{bmatrix}U_1(s)\\ U_2(s)\end{bmatrix}$$

State-space and transfer function modeling equivalence

Example:

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{s+2}{s^2+3s+2} & \dfrac{1}{s+1} \\ \dfrac{2}{s+2} & \dfrac{s}{s+3} \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

```
% Define Laplace variable
s = tf('s');

% Define each element of the 2x2 transfer matrix
G11 = (s + 2)/(s^2 + 3*s + 2);
G12 = 1/(s + 1);
G21 = 2/(s + 2);
G22 = s/(s + 3);

% Build the 2x2 transfer matrix
G = [G11 G12; G21 G22];

% Convert the transfer matrix to state-space realization
sys_ss = ss(G);

% Display the result
disp('State-space realization:')
sys_ss
```

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} -3 & -2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 0 \\ 2 & 0 \\ 0 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0.5 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

State-space and transfer matrix modeling equivalence

Note:

- Having a model in frequency domain (transfer matrix) does not imply that stability analysis must also be performed in the frequency domain. A state space realization can be derived for time domain stability studies

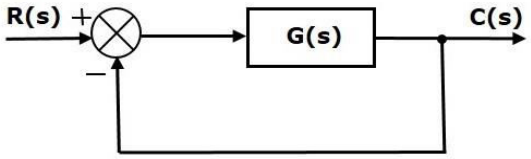
- The A-matrix dimension is typically larger than the dimension of the transfer-matrix G(s)

However:

- The transfer-matrix entries are rational polynomials in the s-domain, and the complexity of stability analysis grows rapidly with system dimension.

- State-space has robust, scalable numerical tools for stability analysis. Transfer-matrix computations in the s-domain become algebraically complex and numerically fragile as system size grows.

Stability test in SISO systems



$$\frac{G(s)}{1+G(s)} = 1 - \frac{1}{1+G(s)}$$

If the goal is merely stability test, it can be done directly using methods such as Routh–Hurwitz Criterion

Nyquist plot is NOT needed

Example:

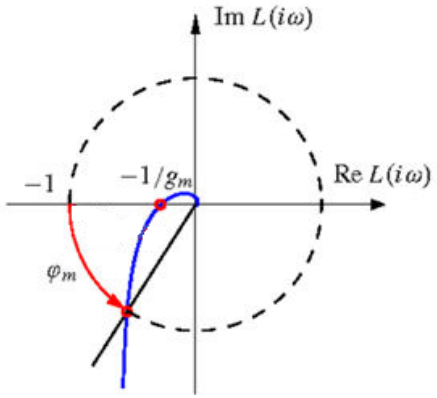
$$G(s) = \frac{4(s+2)}{(s+3)(s+5)}$$

$$1 + G(s) = (s+3)(s+5) + 4(s+2) = s^2 + 8s + 19 = 0$$

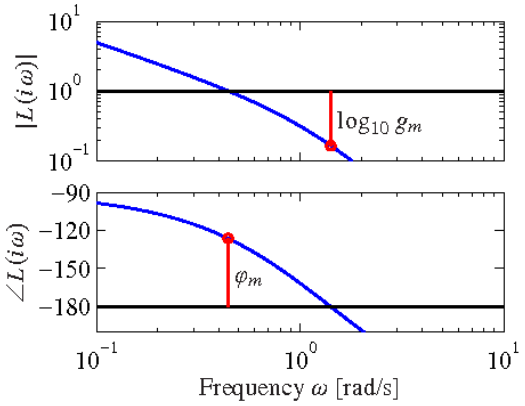
Applying the Routh–Hurwitz Criterion to 1+G(s), we can conclude all poles in the left-half plane and the system is stable.

Robustness assessment in SISO systems using phase and gain margins

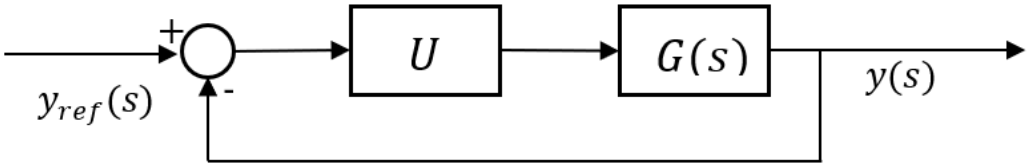
Phase and gain margins are commonly used for robustness assessment in SISO systems



(a) Nyquist plot



(b) Bode plot

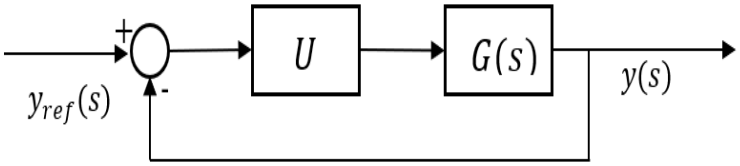


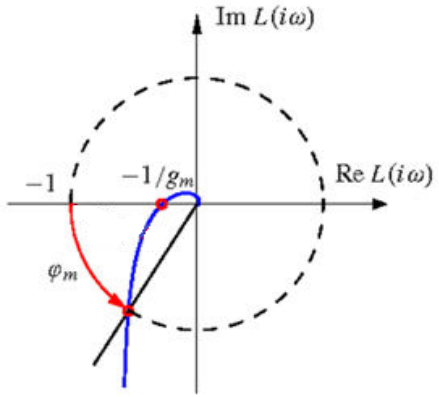
$$U = ke^{j\theta}$$

Gain margin: scaling by the gain k while θ=0

Phase margin: Rotating by the phase angle θ while k=1.

Robustness assessment in SISO systems using phase and gain margins





(a) Nyquist plot

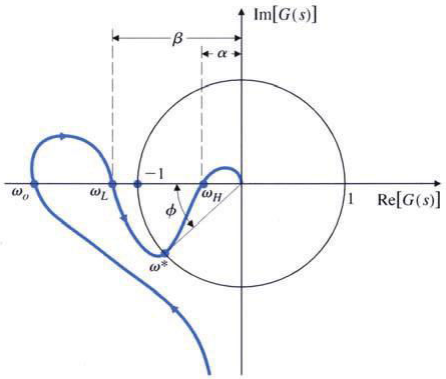
In the phase and gain margins we want to determine number of RHP zero of

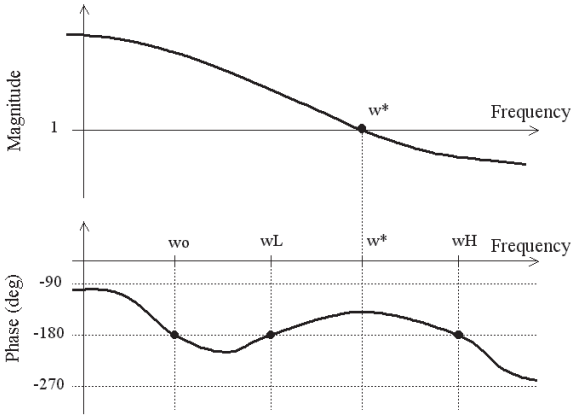
$$1 + kG(j\omega) \quad (1)$$

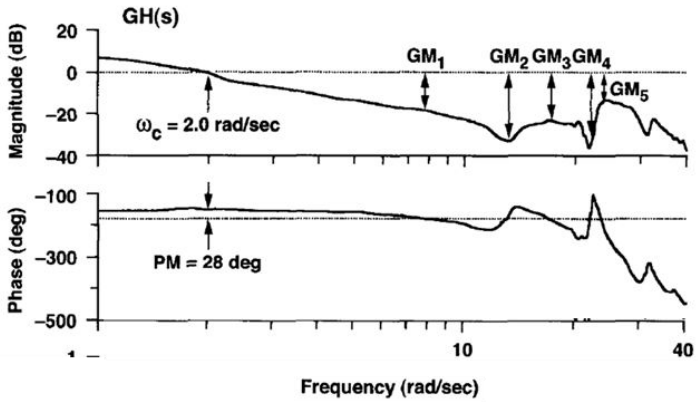
According to the principle of the argument, the number of right half plane (RHP) zeros of 1+kG(s) can be determined by plotting the Nyquist diagram of 1+kG(jw) and counting its encirclements of the origin.

However, since 1+G(jw) is simply G(jw) shifted by –1 in the complex plane, it is more convenient to plot G(jw) instead. In this way, the effect of the gain k can be studied easily by simply expanding or contracting the Nyquist plot of G(jw)

Robustness assessment in SISO systems using phase and gain margins





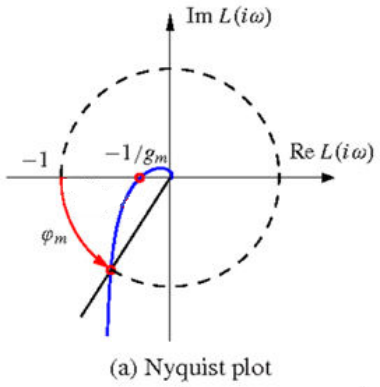


- Bode plot in SISO systems is essentially an alternative representation of the Nyquist plot conveying the same stability information but using logarithmic frequency on the horizontal axis instead of polar coordinates. While the Bode plot is often more suitable for control design, the Nyquist plot is generally more straightforward for stability analysis. For instance, in a Nyquist plot the behavior at infinity is naturally incorporated, since the closed contour explicitly includes the point at infinity and its mapping is directly visible in the complex plane.

- The phase/gain margins defined in the commonly known “Bode stability test” are a special case of the Bode plot, applicable only to open-loop stable systems with a single crossover frequency. In general, to assess the stability of a SISO system using the Bode plot, the number of crossover frequencies should be counted, and the number of encirclements calculated. This corresponds to Nyquist plots of conditionally stable systems, where the presence of multiple crossover frequencies can still lead to a stable system depending on the number of encirclements

Robustness assessment in SISO systems using phase and gain margins

Disk margin: Disk margin is used which considers simultaneous phase and gain perturbations in contrast to Phase and gain margins consider the phase and gain perturbations one at a time.



(a) Nyquist plot

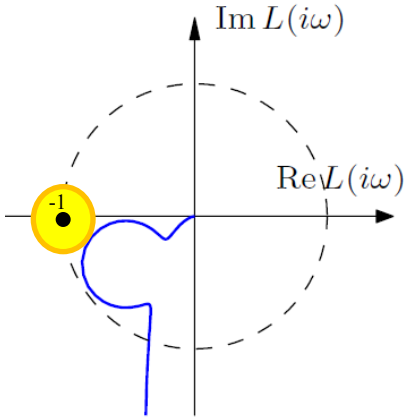
- Example:

$$L(s) = \frac{0.38(s^2 + 0.1s + 0.55)}{s(s+1)(s^2 + 0.06s + 0.5)}$$

Gain margin = 266
Phase margin:70◦.

the system is robust,

but disc margin is small 0.27, which is very low.

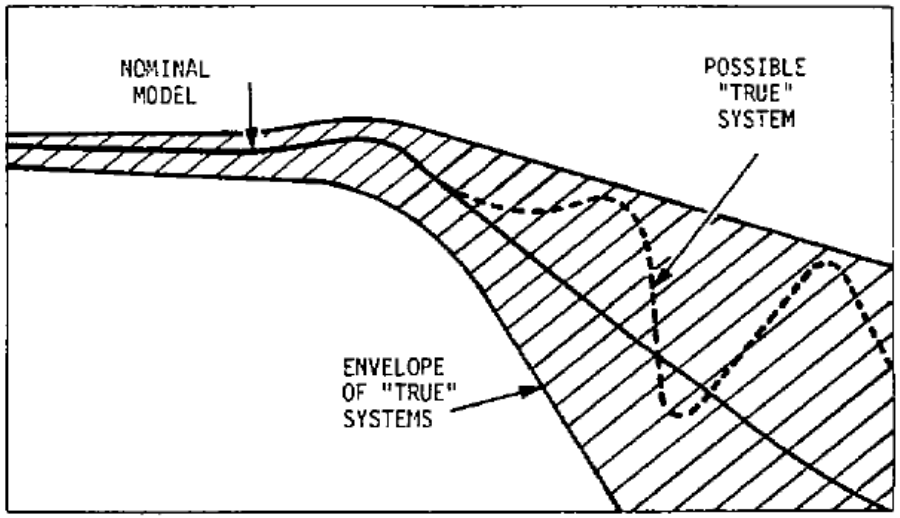


- However, this happens in very rare situations

K. J. Åström and R. M. Murray, *Feedback Systems: An Introduction for Scientists and Engineers*, 2nd ed. Princeton, NJ: Princeton University Press, 2021.

Advantages of phase and gain margin indicators

- They are reliable measure of robustness in SISO systems
- It is a strong tool for controller design through loopshaping.



Advantages of phase and gain margin indicators

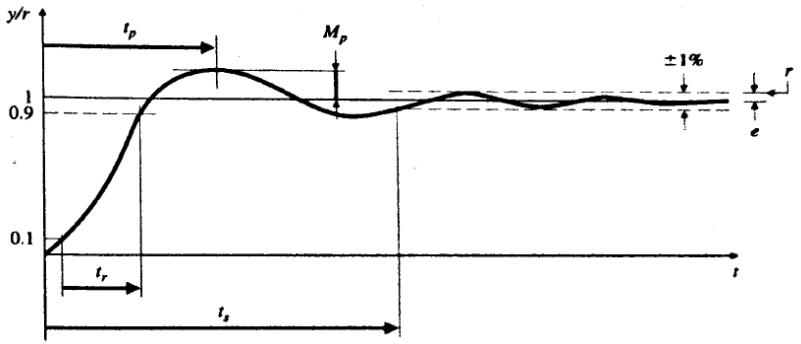
- In SISO systems gain and phase margins are directly linked to controller performance and are commonly used as design criteria

$$M_p = 5\% \; \zeta = 0.7$$

$$M_p = 15\% \; \zeta = 0.5$$

$$M_p = 35\% \; \zeta = 0.3$$

$$\zeta \cong \frac{PM}{100} \; for \; PM < 70^\circ$$



$$M_p \leq 10\% \Rightarrow \zeta \approx 0.6 \Rightarrow Phase\ margin\ 60^\circ$$

Unfortunately, the above advantages do not apply to MIMO systems

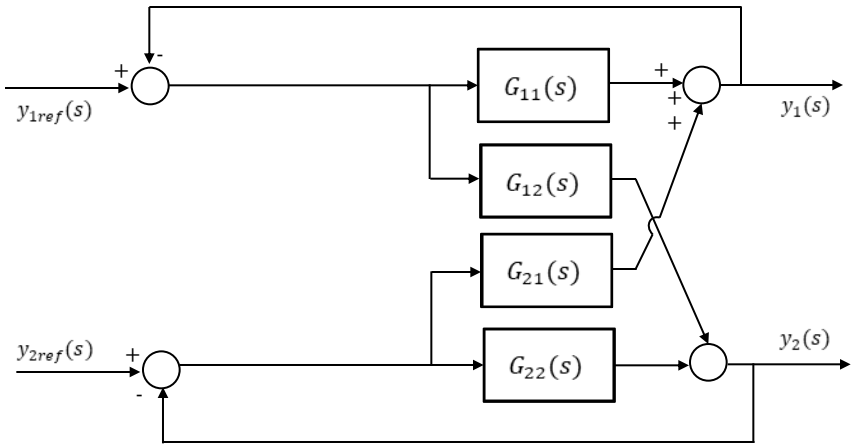
G. F. Franklin, J. D. Powell, and A. Emami-Naeini, *Feedback Control of Dynamic Systems*, 7th ed. Pearson, 2014.

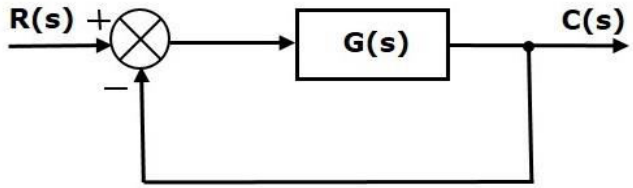
Key Takeaways

Nyquist plots (SISO)

- **Stability test:** It provides correct results, but its intended application is not stability test as much more efficient methods exist
- **Robustness test:** They are powerful tools for robustness analysis and controller design

Stability test in MIMO systems

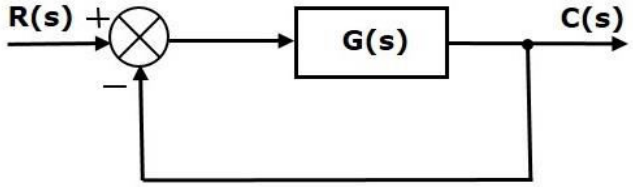




$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

If the goal is merely stability analysis, one can directly determine the stability, no Nyquist criterion or bode criterion is needed

Stability test in MIMO systems



$$G(s)\big(I+G(s)\big)^{-1} = I - \big(I+G(s)\big)^{-1}$$

Nyquist plot is NOT needed

Example:

$$G(s) = \begin{bmatrix} \dfrac{2}{s+1} & \dfrac{1}{s+2} \\ \dfrac{1}{s+3} & \dfrac{3}{s+1} \end{bmatrix}$$

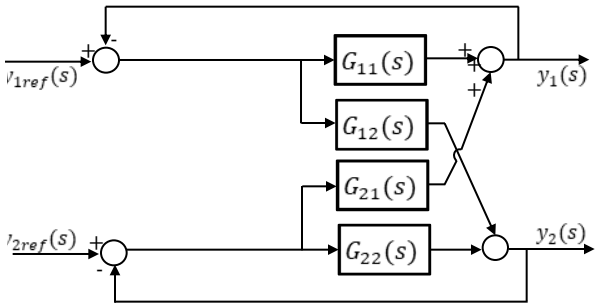
Check det(I+G(s)) if does not have RHP zeros

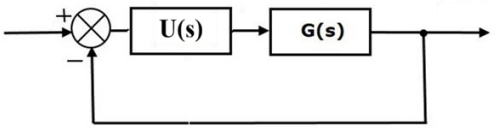
$$s^4 + 9s^3 + 31s^2 + 49s + 32$$

Applying the Routh–Hurwitz Criterion all zeros are in the left-half plane.

- Note in the IBR stability analysis we assume G(s) is stable.

Robustness assessment in MIMO systems





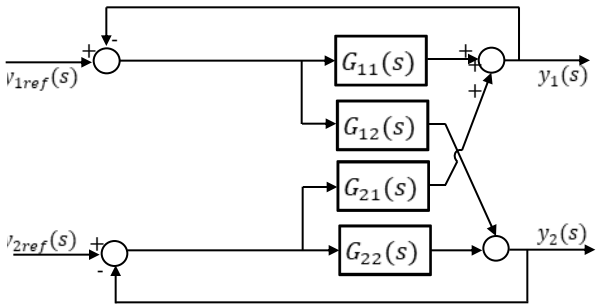
$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

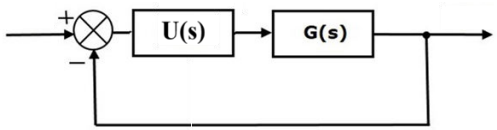
- Early attempts in the 1970s focused on extending the concepts of phase margin and gain margin from SISO systems to MIMO systems

  - An early approach was based on the 'single-loop-at-a-time' concept. However, it is unrealistic to assume that disturbances do not occur simultaneously in different channels.

- The Generalized Nyquist Criterion (GNC) amid at considering multi-loop feedback structure of MIMO systems

Generalized Nyquist Criterion





$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

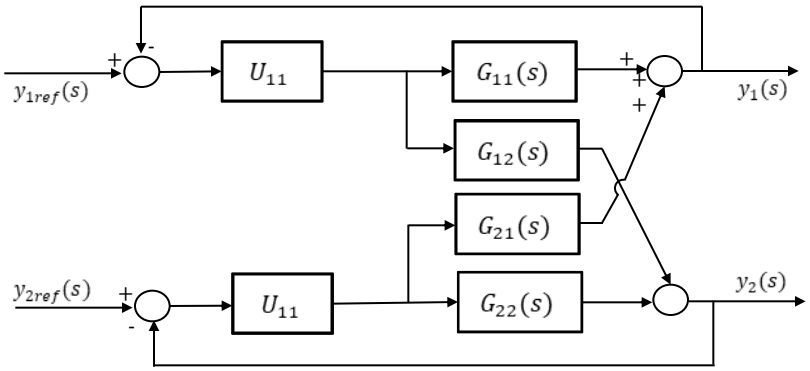
$$U(s) = \begin{bmatrix} U_{11}(s) & U_{12}(s) \\ U_{21}(s) & U_{22}(s) \end{bmatrix}$$

The GNC is based on very restrictive assumptions and applies only to a very limited class of uncertainty in the form of

$$U_{11} = U_{22} = e^{j\theta} \text{ and } U_{12} = U_{21} = 0$$

$$U = \begin{bmatrix} ke^{j\theta} & 0 \\ 0 & ke^{j\theta} \end{bmatrix} = ke^{j\theta} I$$

known as uniform uncertainty



Generalized Nyquist Criterion

$$Det\left(I + G(s)U(s)\right)$$

$$det\left(I + G(j\omega)U(j\omega)\right) =$$
$$det\left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + \begin{bmatrix}G(j\omega)_{11} & G(j\omega)_{12}\\ G(j\omega)_{21} & G(j\omega)_{22}\end{bmatrix}\begin{bmatrix}ke^{j\theta} & 0\\ 0 & ke^{j\theta}\end{bmatrix}\right)$$

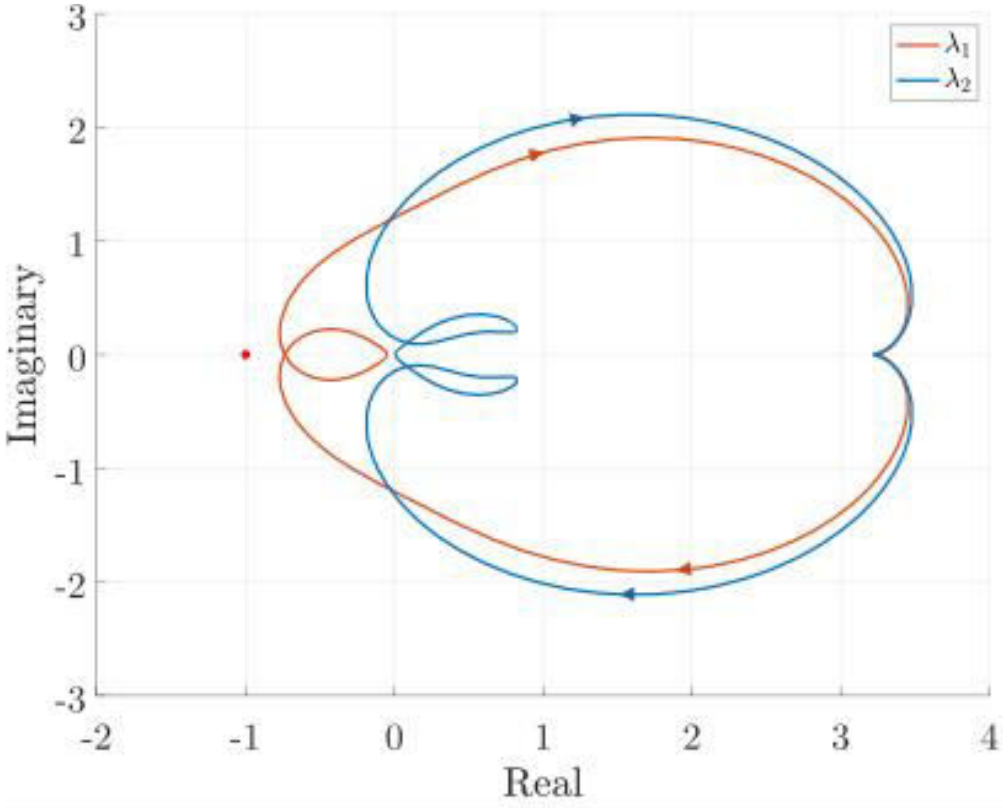
$$det\left(I + G(j\omega)U(j\omega)\right) =$$
$$\prod_{i=1}^{2}\lambda_i(1 + \left(G(j\omega)ke^{j\theta}I\right) = \prod_{i=1}^{2}(1 + ke^{j\theta}\lambda_i(G(j\omega)))$$

We know that

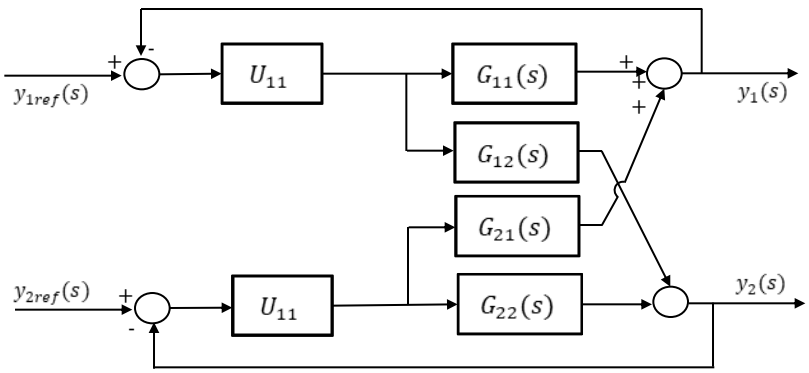
$$\lambda_i(1 + G(j\omega)) = 1 + \lambda_i(G(j\omega))$$

The problem is reformulated as counting the encirclements of the critical point -1 by the locus of $ke^{j\theta}\lambda_i\left(G(j\omega)\right)$which can be visualized by plotting the eigenvalue trajectories $\lambda_i\left(G(j\omega)\right)$, scaling them by the gain k, and rotating them by the phase angle θ.

Generalized Nyquist Criterion



Generalized Nyquist Criterion



$$G(s)=\begin{pmatrix} \frac{1}{s+1}+\frac{b}{s+2} & \frac{b}{s+2} \\ -\frac{b}{s+2} & \frac{1}{s+1}-\frac{b}{s+2} \end{pmatrix}$$

$$U=\begin{pmatrix} k_1 & 0 \\ 0 & k_1 \end{pmatrix}$$

Gain Margin: the closed-loop system is stable for any positive constant $k1$

$$U=\begin{pmatrix} e^{j\theta_1} & 0 \\ 0 & e^{j\theta_1} \end{pmatrix}$$

Phase Margin: the closed-loop system is stable provided $|\theta| < \frac{3\pi}{4}$

$$U=\begin{pmatrix} \frac{b-4}{b+4} & 0 \\ 0 & 1 \end{pmatrix}$$

- the closed-loop system becomes unstable which is clearly a very small deviation from 1 for large value of b.

Generalized Nyquist Criterion

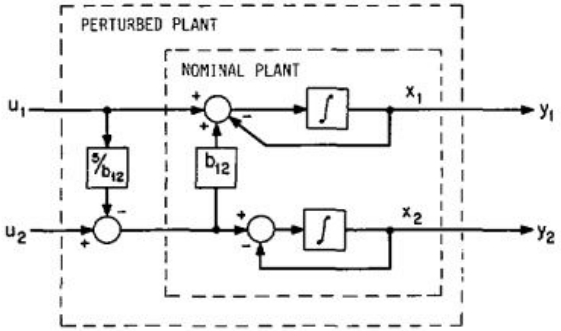


Fig. 4. Perturbation in nominal open-loop system which makes closed-loop system unstable.

$$G(s)=\begin{pmatrix} \frac{1}{s+1} & \frac{b}{s+1} \\ 0 & \frac{1}{s+1} \end{pmatrix}$$

$$U=\begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}$$

*Gain Margin: the closed-loop system is stable for any positive constants k*1, k2.

$$U=\begin{pmatrix} e^{j\theta_1} & 0 \\ 0 & e^{j\theta_2} \end{pmatrix}$$

Phase Margin: the closed-loop system is stable provided. $\theta_1, \theta_2 < \frac{3\pi}{4}$

$$U = \begin{pmatrix} 1 & 0 \\ -\frac{5}{b} & 1 \end{pmatrix}$$

which is clearly a very small perturbation for b large. This example illustrates that it is dangerous to consider gain and phase margin as a robustness measure for MIMO systems:

N. Lehtomaki, N. Sandell, and M. Athans, "Robustness results in linear-quadratic Gaussian based multivariable control designs," *IEEE Trans. Autom. Control*, vol. 26, no. 1, pp. 75–93, Feb. 1981.

Eigenvalue is not a reliable indicator of proximity to singularity

Example:

$$A = \begin{bmatrix} 1 & \frac{1}{\varepsilon} \\ 0 & 1 \end{bmatrix}$$

$\lambda_1 = \lambda_2 = 1$ independent of ε far from zero, suggesting not close to singularity (perfectly well-conditioned). However, is only an ε away from being singular!

$$\varepsilon = 0.001 \qquad A = \begin{bmatrix} 1 & 1000 \\ 0 & 1 \end{bmatrix}$$

Instead of 0 put $\varepsilon = 0.001$ $(i.e. has\ a\ small\ uncertainty)$

$$A = \begin{bmatrix} 1 & 1000 \\ 0+\varepsilon & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1000 \\ 0.001 & 1 \end{bmatrix}$$

Det (A)=0. It is singular! (it was only an ε away from being singular)

$$\sigma_{max} = 1000.001 \quad \sigma_{min} = 0.001$$

Condition number $\kappa(A) = \frac{\sigma_{max}}{\sigma_{min}} \approx 10^6$ meaning extremely ill-conditioned

B. Friedland, Control System Design: An Introduction to State-Space Methods. Dover Publications, 2005.

Phase and gain margins in MIMO systems

- To address the problem of GNC, singular value-based method was proposed.
- It provides extremely conservative results.

$$U(s) = \begin{bmatrix} U_{11}(s) & U_{12}(s) \\ U_{21}(s) & U_{22}(s) \end{bmatrix}$$

This is because they allow for the simultaneous magnitude and phase variations of ***any*** of the gains

$$\mathrm{GM} = \frac{1}{1 \pm \alpha_0}$$

$$\mathrm{PM} = \pm \cos^{-1}\left[1 - \frac{\alpha_0^2}{2}\right]$$

$$\underline{\sigma}\big(\mathbf{I} + \mathbf{G}(s)\big) \geq \alpha_0$$

We applied it to a system of single IBR to infinite bus

PM based on above equations: 8.53

- Too conservative, but reliable

PM based on GNC: 66.10

- Not conservative, but unreliable

N. Lehtomaki, N. Sandell, and M. Athans, "Robustness results in linear-quadratic Gaussian based multivariable control designs," *IEEE Trans. Autom. Control*, vol. 26, no. 1, pp. 75–93, Feb. 1981.

Phase and gain margins in MIMO systems

Extensive work were carried out in 1980 and 1990

$$U=\begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}$$

- For example, some work tried to enforce the structure of non-uniform gain margin and formulated the problem as an optimization problem. However, the underlying optimization problems are non-convex, which introduces the risk of convergence to local minima. As a result, the computed gain and phase margins may not reflect the true worst-case margins, smaller perturbations capable of destabilizing the system may exist but remain

- Some work introduce matrix-based representations of gain and phase margins. They argue that, in SISO systems, gain and phase margins can be represented as $ke^{j\theta}$, where k is a positive scalar (analogous to a positive definite matrix in MIMO systems), and $e^{j\theta}$ captures the phase rotation (analogous to a unitary matrix in MIMO systems). One major drawback is that the results may lack physical interpretability, potentially leading to nonrealistic scenarios that are unlikely to occur in practical systems.

Phase and gain margins in MIMO systems

- Specially in IBR stability analysis, G(s)=Y(s)Z(s). It is impossible to incorporate information about uncertainties in Y and Z separately into the studies

- After extensive studies in the 1980s and 1990s, it was concluded that robustness measures such as phase and gain margins used for SISO systems cannot be directly extended to MIMO systems. Since then, attention has shifted to modern robustness assessment methods, such as μ-analysis.

Key Takeaways

Generalized Nyquist Criterion

Stability test: It provides correct results, but its intended application is not stability test as much more efficient methods exist

Robustness test: DO NOT use it as it provides unreliable results

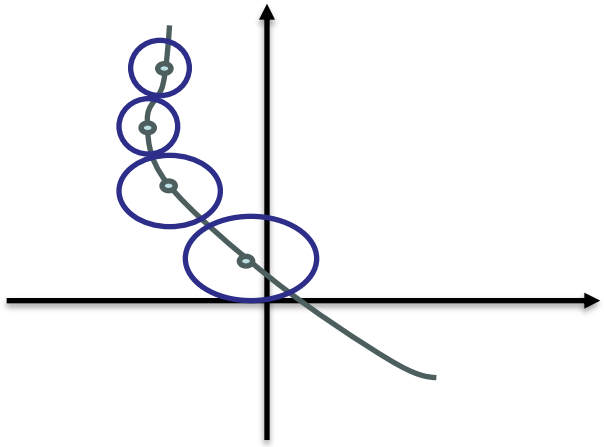
Gershgorin approximation

Let A=$[a_{ij}]$ is an n×n matrix. For each i=1,2,..,n define the Gershgorin disc

$D_i = \{z \in \mathbb{C} \mid |z - a_{ii}| \leq R_i\}$ where $R_i = \sum_{j \neq i} |a_{ij}|$

Then, every eigenvalue of A lies within the union of these discs

$$\lambda(A) \subseteq \bigcup_{i=1}^{n} D_i$$

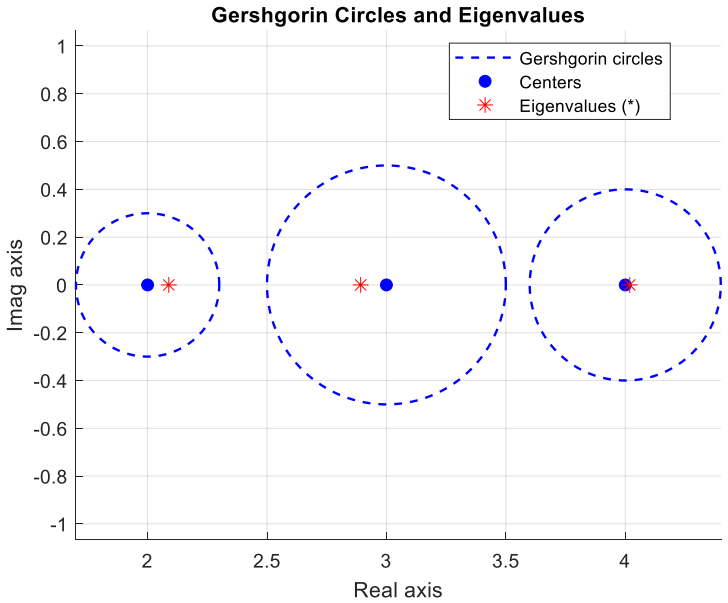


$$A = \begin{bmatrix} 3 & 0.5 & 0 \\ -0.2 & 2 & 0.1 \\ 0 & 0.4 & 4 \end{bmatrix}$$

$$\lambda_1 = 2.09$$

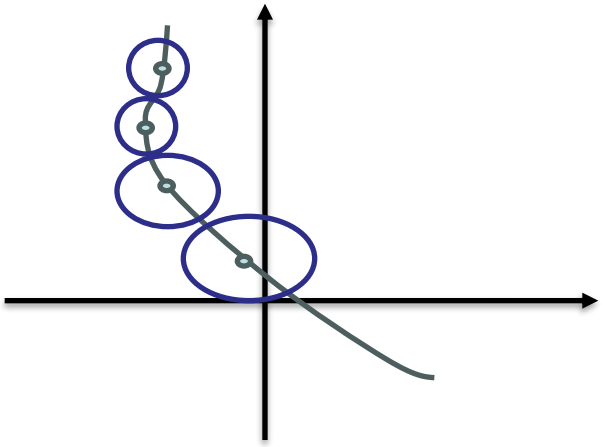
$$\lambda_2 = 2.89$$

$$\lambda_3 = 4.02$$



Gershgorin approximation

- Variety of methods have been proposed based on Gershgorin disc concept such as G-norm



- G norm: In the 2*2 matrix of YZ, restricting the larges
than 0.5 for all frequencies will ensures eigenvalues of

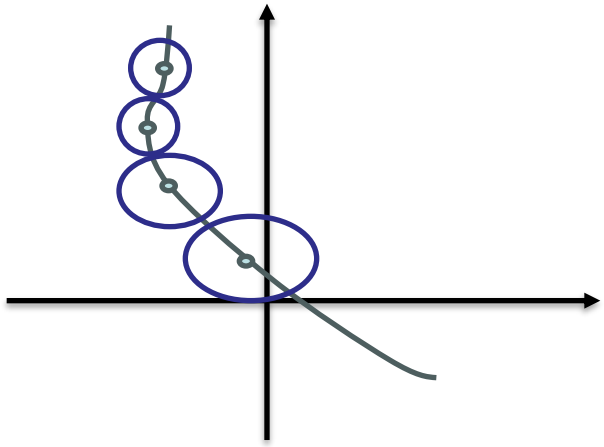
$$\|G(j\omega)\|_G = \|Y(j\omega)Z(j\omega)\|_G$$
$$= \max(|G_{11}(j\omega)|, |G_{12}(j\omega)|, |G_{21}(j\omega)|, |G_{22}(j\omega)|)$$

$$\|Y(j\omega)Z(j\omega)\|_G < 0.5 \; for - \infty < \omega < \omega$$

$$\|Y(j\omega)\|_G \|Z(j\omega)\|_G < 0.5$$

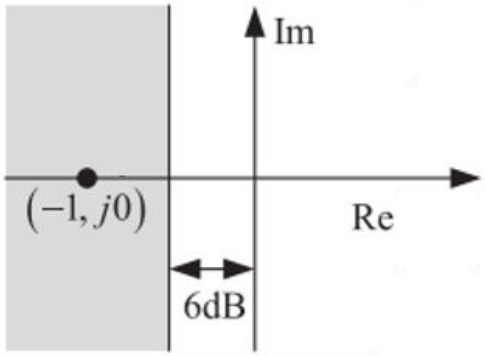
Gershgorin approximation

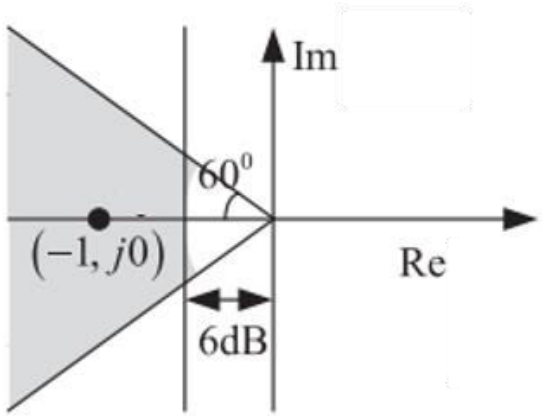
- This method produces useful results only if diagonal dominance condition is satisfied, otherwise it give a very conservative results.
- A MIMO system with n channels and a diagonally dominant transfer matrix can be views as n independent SISO systems, with only minor interactions between the channels.



- Methods like Gnorm, requiring the magnitude of eigenvalues remain within a unit circle significantly increase conservativeness
- Using the inequality relation adds another layer of conservatism
- These approaches may be used to provide some insights on influential factors (design oriented approach), but may provide extremely conservative results that become practically useless for stability analysis

Methods relying on GNC





- Nyquist criterion is reliable for SISO systems. Therefore, Forbidden region methods for SISO systems and can be used

- GNC for MIMO system is not a reliable for robustness assessment. Therefore, any other methods such as forbidden region methods relying on GNC are not reliable too.

Small gain theorem

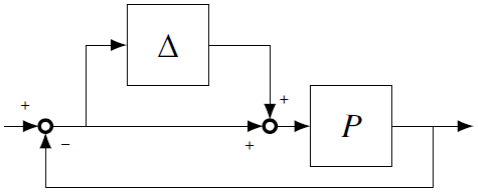


Fig. 1. Multiplicative uncertainty

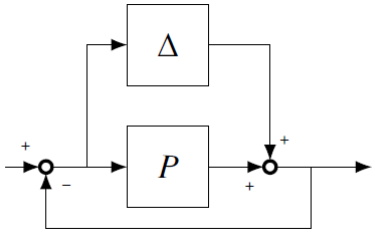


Fig. 2. Additive uncertainty

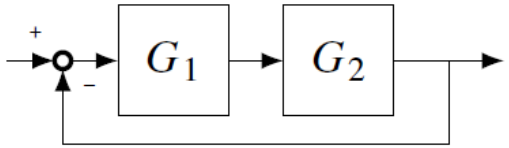


Fig. 3. Standard interconnection of two stable systems G1 and G2 in small gain theorem

$\Delta = U - I$ is norm bounded

In multiplicative uncertainty: $G_1 = \Delta \; and G_2 = P(I+P)^{-1}$

In additive uncertainty: $G_1 = \Delta \; and G_2 = (I+P)^{-1}$

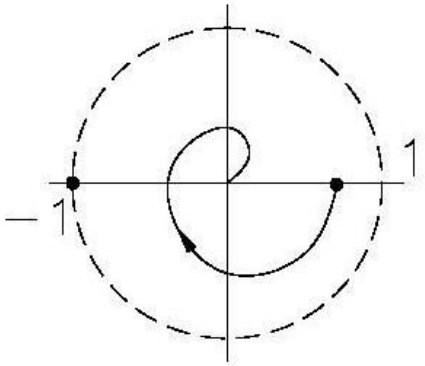


Fig. 4. $\|G\|_\infty$<1

Fig. 3 is stable for two stable systems G_1 and G_2 if $\|G_1 G_1\|_\infty$<1.

$$\|G_1 G_2\|_\infty \leqslant \|G_1\|_\infty \|G_2\|_\infty$$

In IBR stability analysis $G_1 = Z$ and $G_2 = Y$. $\|YZ\|_\infty < 1$

Note: In the robustness analysis $G_1 = \Delta$ is assumed bounded unknown. However, in IBR stability analysis both $G_1 = Z$ and $G_2 = Y$ are known and one can just check the stability of the closed loop system directly without invoking the small gain theorem which is extremely conservative (look at Fig. 4)

Small phase theorem

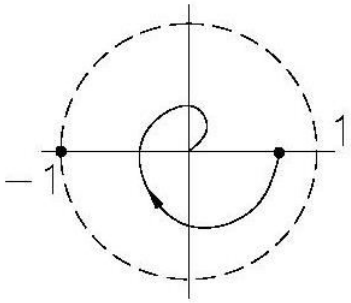


Fig.1 Small gain theorem

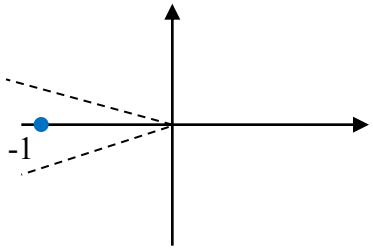


Fig.2 Small phase theorem

- In the small phase theorem, the system's phase is constrained to avoid crossing the ±180° boundary, thereby ensuring that the Nyquist plot does not intersect the negative real axis.
- Similarly, the small gain theorem requires the system's gain to remain less than one, ensuring that there is no encirclement of the critical point at -1
- The system model tends to become less accurate at higher frequencies due to physical limitations of measurement equipment, parameter estimation errors, or the presence of unmodeled dynamics.
- In well-designed systems, however, the magnitude of the frequency response naturally rolls off as frequency increases, resulting in smaller gains. Consequently, the small-gain theorem becomes particularly effective. However, small phase theorem becomes less reliable as the phase angle calculation becomes highly sensitive to modeling inaccuracies in the high frequency range where Yinv exhibits a strong roll-off

Determinant based method

The determinant-based method states that, unlike the Generalized Nyquist Criterion (GNC) which requires plotting the Nyquist diagrams of multiple eigenvalues, we can simply draw the Nyquist plot of

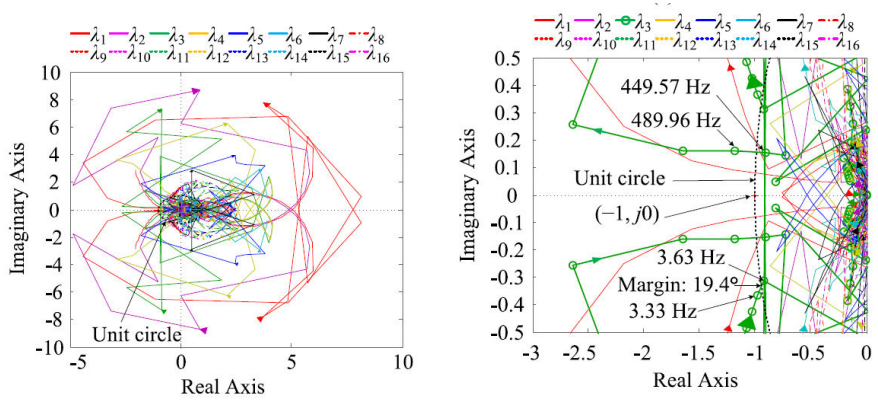
$$Det(I + G(j\omega))$$

and count the number of encirclement of the origin in instead of -1

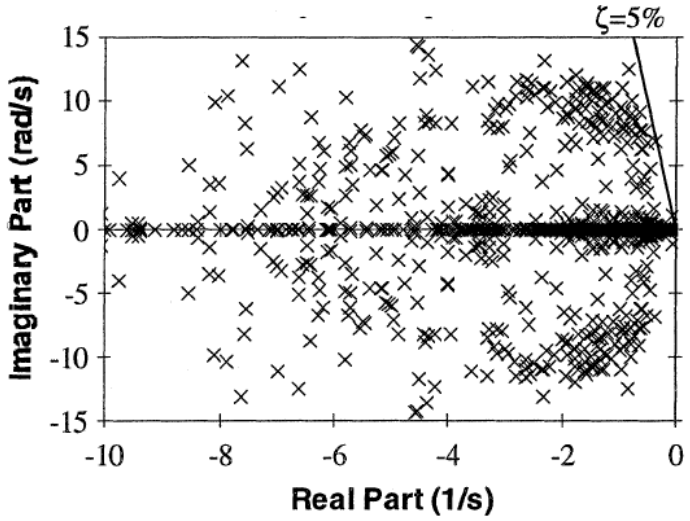
- As discussed earlier, the primary goal of the GNC is robustness analysis. The use of the Nyquist plots of eigenvalues, rather than the Nyquist plot of the determinant, was motivated by the desire to define phase and gain margins in a manner analogous to SISO systems. The GNC was not intended to count the number of right-half-plane zeros of $Det(I + G(j\omega))$

Multi machine

- Convoluted GNC plots makes it difficult to scale it to large systems



- Small signal security assessment is also needed beyond the stability assessment, meaning a typical damping ratio of 5% on eigenvalues. Participation factors calculation is also needed.



S. Gomes, Jr., N. Martins, and C. Portela, "Computing small-signal stability boundaries for large-scale power systems," IEEE Transactions on Power Systems, vol. 18, no. 2, pp. 747–752, May 2003

W. Cao, Y. Ma, F. Wang, L. M. Tolbert, and Y. Xue, "Low-Frequency Stability Analysis of Inverter-Based Islanded Multiple-Bus AC Microgrids Based on Terminal Characteristics," IEEE Transactions on Smart Grid, vol. 11, no. 5, pp. 3662–3676, Sep. 2020.

Multi-machine

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) & \dots \\ G_{21}(s) & G_{22}(s) & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

- Directly finding the poles from the transfer matrix is challenging (numerical complexity and catastrophic coefficient sensitivity). Small coefficient errors can move poles noticeably, particularly if some poles are clustered or near the imaginary axis.

- Calculating eigenvalues of A-matrix is computationally more robust and more efficient.

Passivity

A SISO transfer function G(s) is passive if it is stable and positive real, i.e.

$$\Re\{G(j\omega)\} \geq 0 \quad \forall\omega \qquad (1)$$

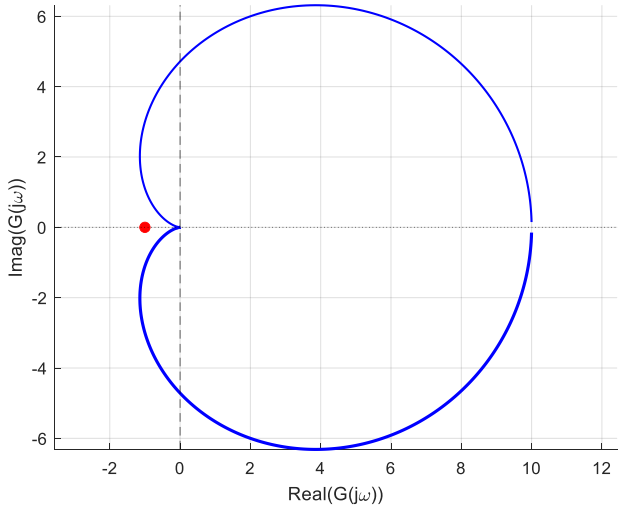
For MIMO systems, transfer matrix G(s) is passive if it is stable and

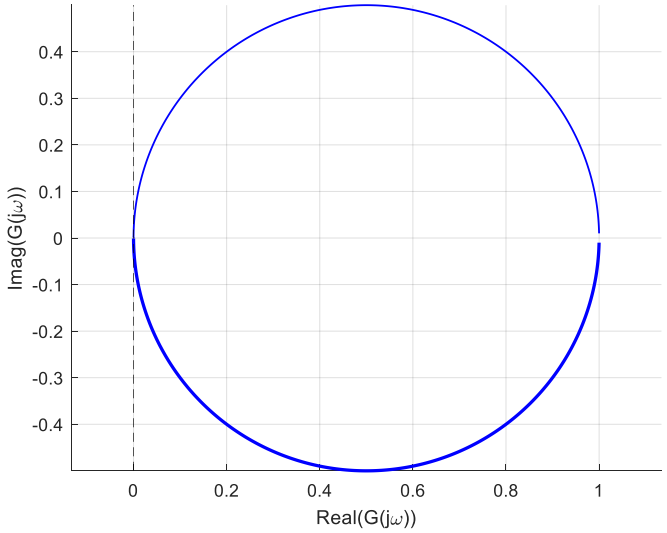
$$G(j\omega) + G(j\omega)^H \succcurlyeq 0 \qquad \forall\omega \qquad (2)$$

- Interconnection of a passive system and a strictly passive system is stable. Since the power network is passive, ensuring the passivity of the IBRs is sufficient for decentralized stability assessment.
- Note at least one of the systems should strictly passive, meaning it is entirely in the open right-half of the complex plane (no crossings or touches of the imaginary axis).

- In many papers, the passivity conditions are evaluated only up to the Nyquist frequency. However, as reported in several papers this procedure can lead to misleading results if not applied carefully, because it may overlook destabilizing interactions that occur beyond the Nyquist frequency if the band-limited plant assumption does not hold

Passivity





$$G(s) = \frac{20}{(s+1)(s+2)}$$

$$\Re\{G(j\omega)\} = \frac{20(2-\omega^2)}{(2-\omega^2)^2+9\omega^2}$$

$$G(s) = \frac{1}{(s+1)}$$

$$\Re\{G(j\omega)\} = \frac{1}{1+\omega^2} \geq 0$$

- This leads to extremely conservative assessment, but has advantages of performing decentralized stability assessment. Unfortunately, IBRs don't meet this requirement

Summary

- The stability analysis of MIMO systems differs fundamentally from that of SISO systems, and SISO-based reasoning cannot be directly applied to MIMO dynamics

- Stability analysis and robustness assessment tools must be used appropriately to avoid overly conservative evaluations, unnecessary computational effort, and potentially misleading or incorrect conclusions..

- The intended application of method should be carefully examined and taken into account

Acknowledgment

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Prof. Ali Saberi,
Washington State University

Prof. Anton A. Stoorvogel,
University of Twente, The Netherlands

Mr. Hassan Yazdani,
Ph.D student Washington State University

Saeed Lotfifard
Washington State University

More details can be found in the following paper:

A. A. Stoorvogel, S. Lotfifard, A. Saberi "On MIMO Stability Analysis Methods Applied to Inverter-Based Resources Connected to Power Systems " arXiv preprint, arXiv:2510.20384, 2025.

Thanks!

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