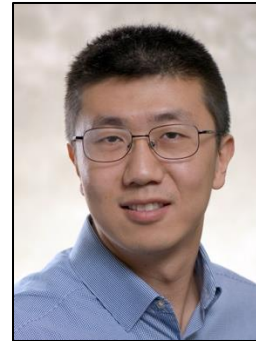


Power System State Estimation by Phase Synchronization and Eigenvectors



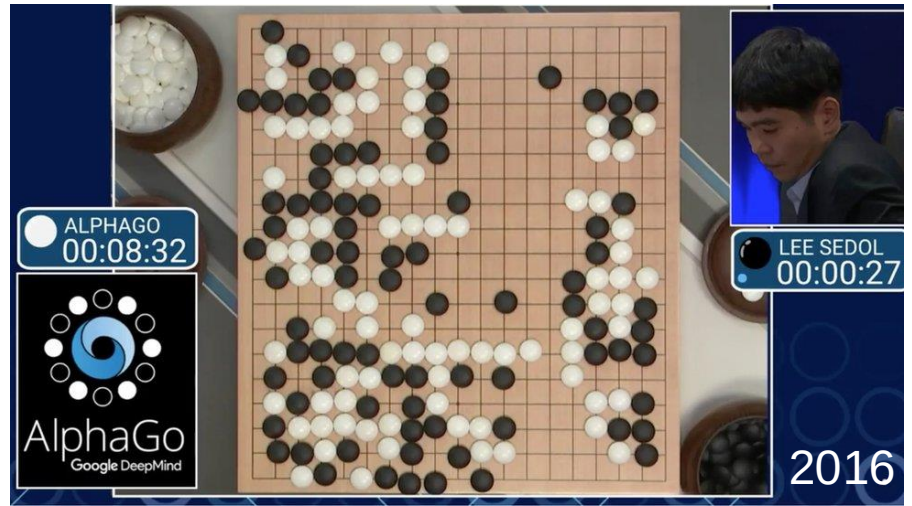
Iven Guzel & Richard Y. Zhang
Univ. of Illinois at Urbana-Champaign

`{iguzel2,ryz}@illinois.edu`

PSERC Webinar
February 12, 2025



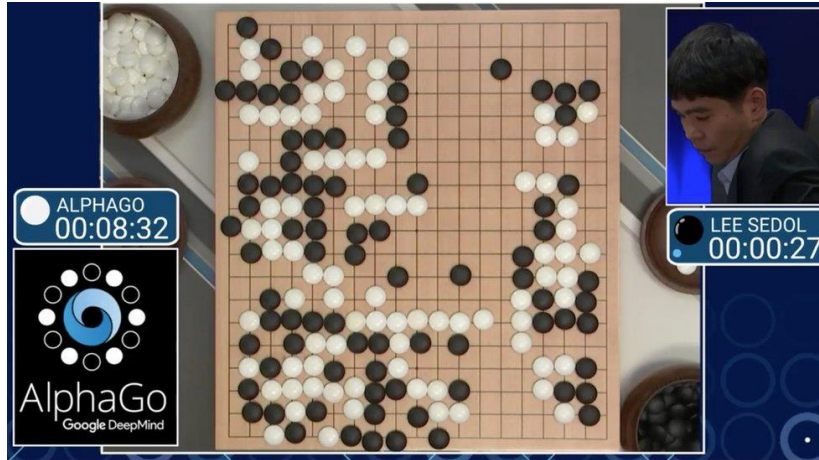
Never shall I be
beaten by a machine!
Can you see a
machine writing a
novel or poetry?
Garry Kasparov (1989)



It may be a hundred
years before a
computer beats
humans at Go –
maybe even longer.
New York Times article (1997)

Can AI be practically useful?

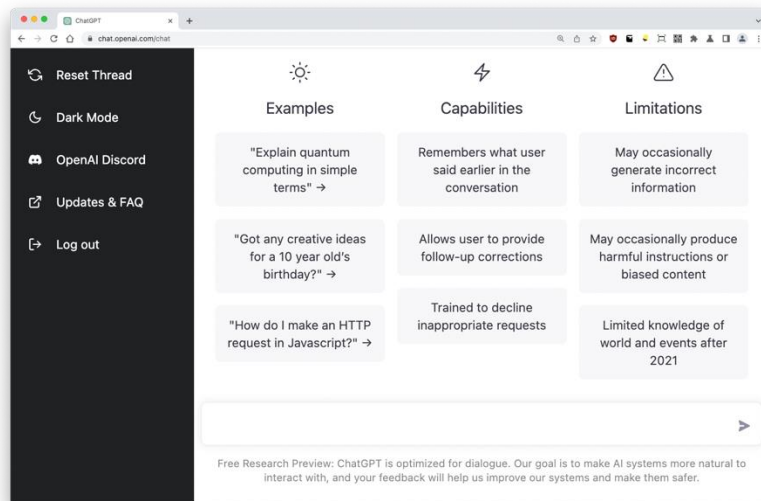
Chess, Shogi, Go (2016)



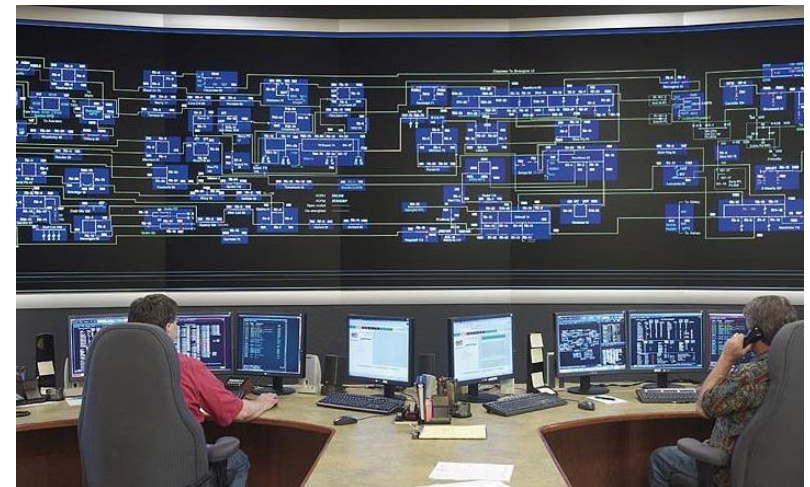
Protein Folding (2024 Nobel)



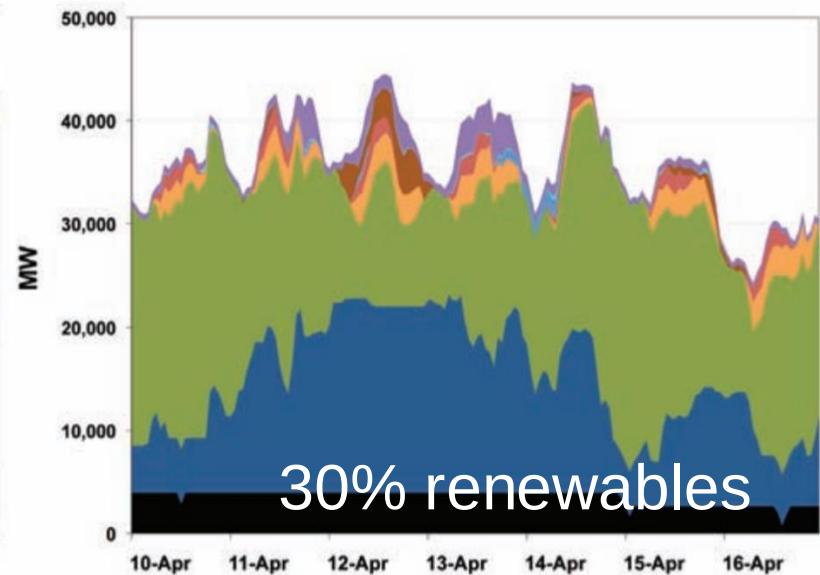
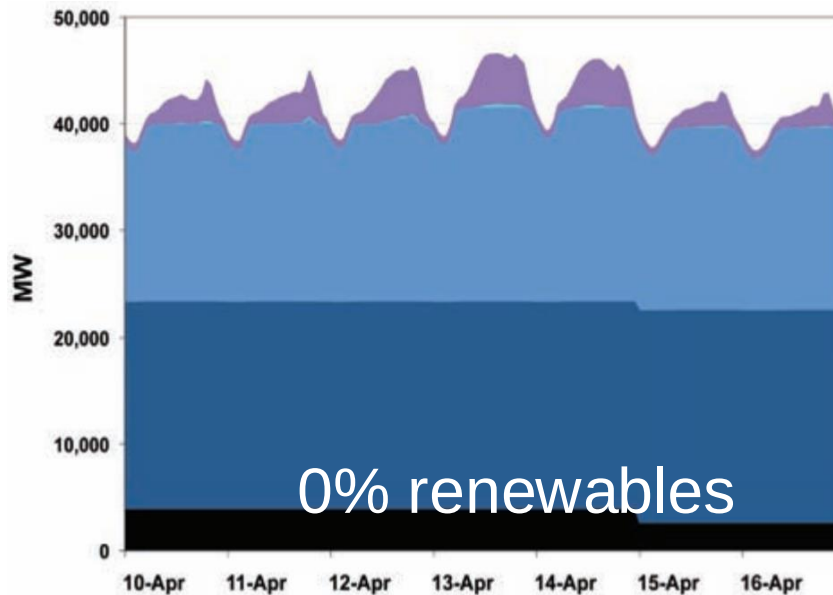
Large Language Models (2022)



Electric grid (????)

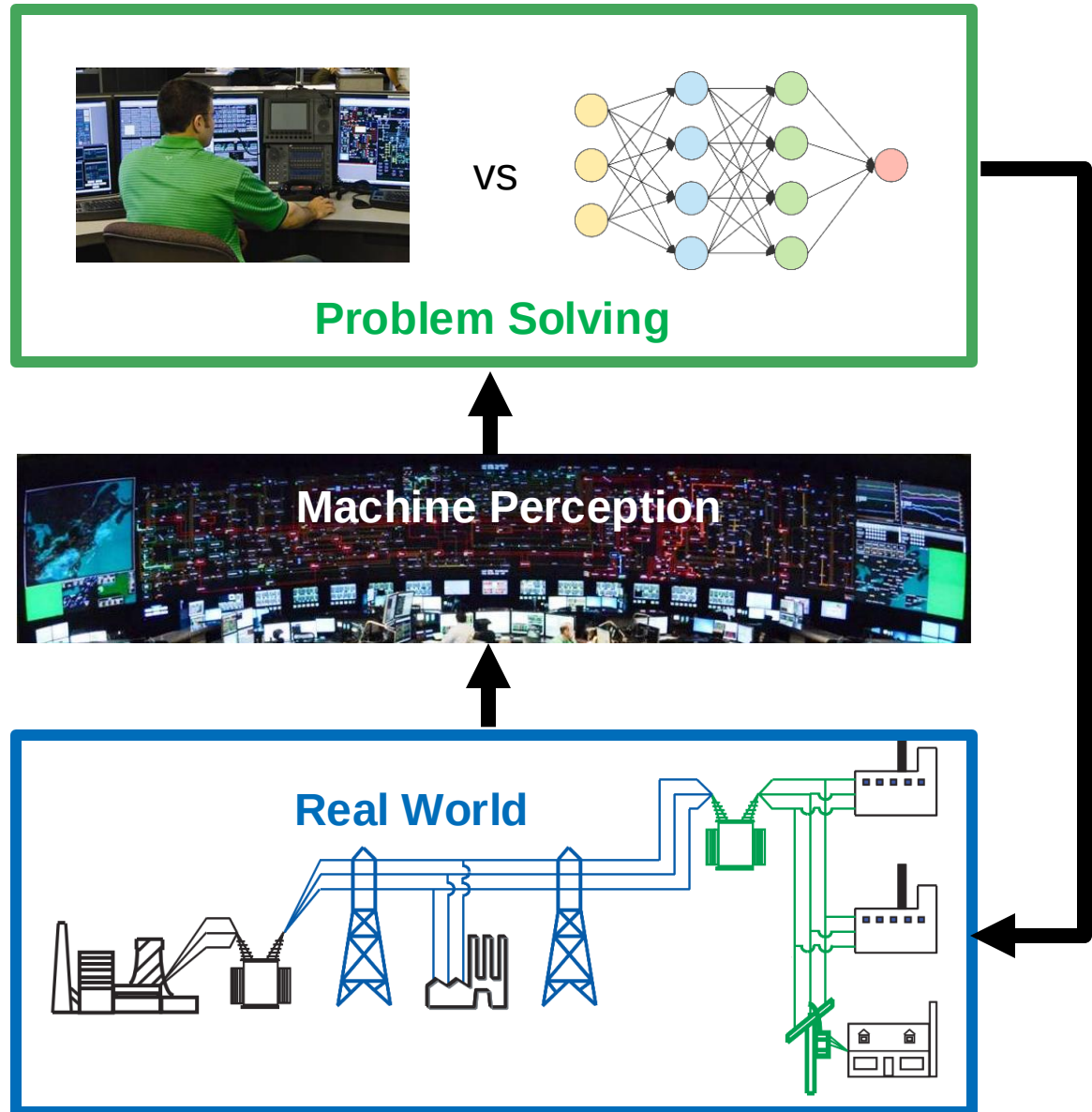


The potential of AI for the electric grid



Source: Western Wind Integration Study

Electric grid is not chess, nor protein folding



Perception

- Sensing
- Estimation
- Inference
- Semantic understanding

Better analogy: Self-driving cars

Waymo/Youtube [youtu.be/2CVInKMz9cA]



Perception is the critical bottleneck

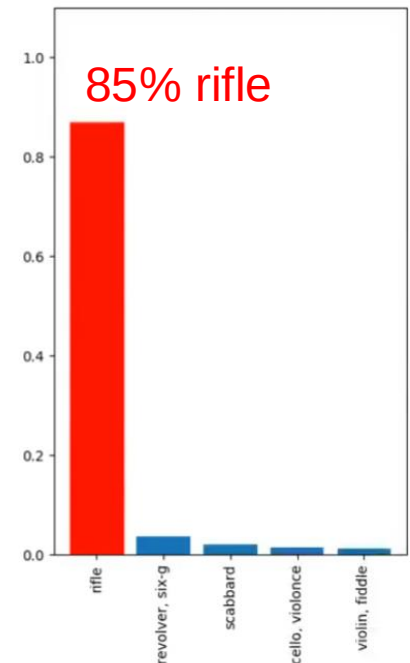
Perception is the critical bottleneck



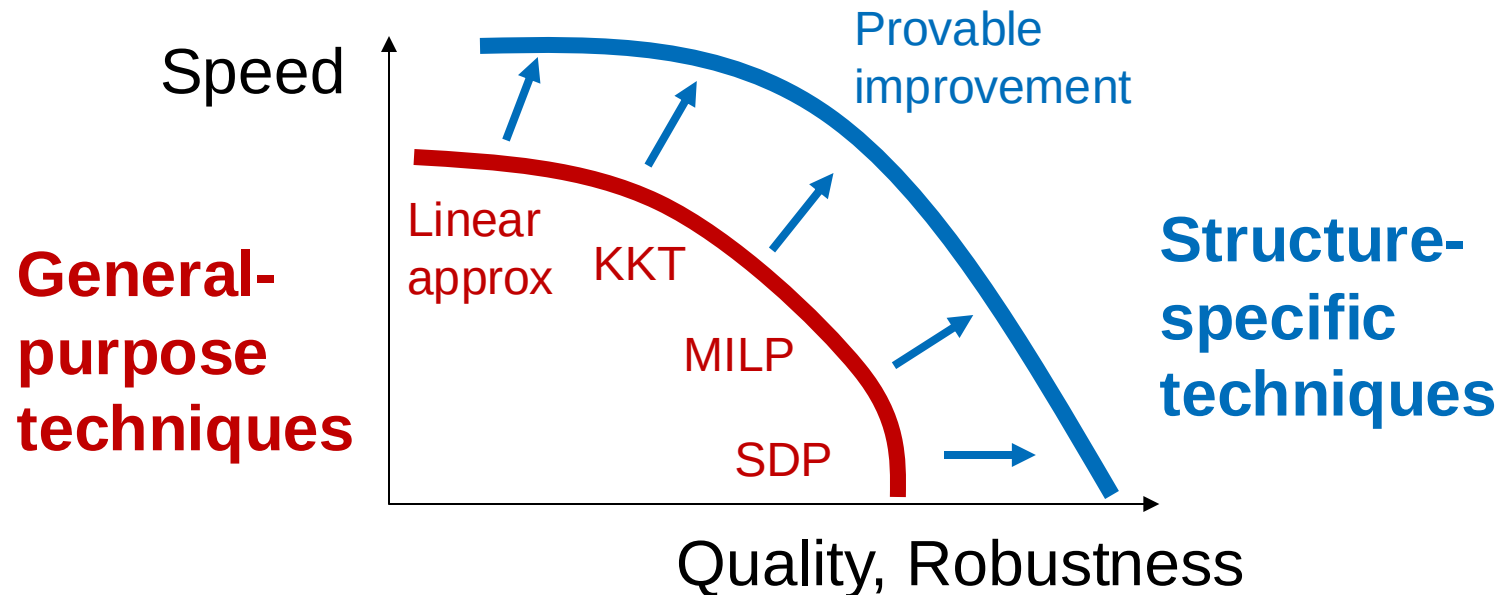
Graffiti causes stop sign to be misclassified as 45mph limit.
[Eykholt et al. 2018]



3D object fools
Google's
InceptionV3
[Athalye 2018]



Our Approach: Structure-Specific Techniques



Provable guarantees on cost, performance, efficiency, reliability, robustness, and safety

Today: Fundamental connection between
grid perception and **phase synchronization**

Outline

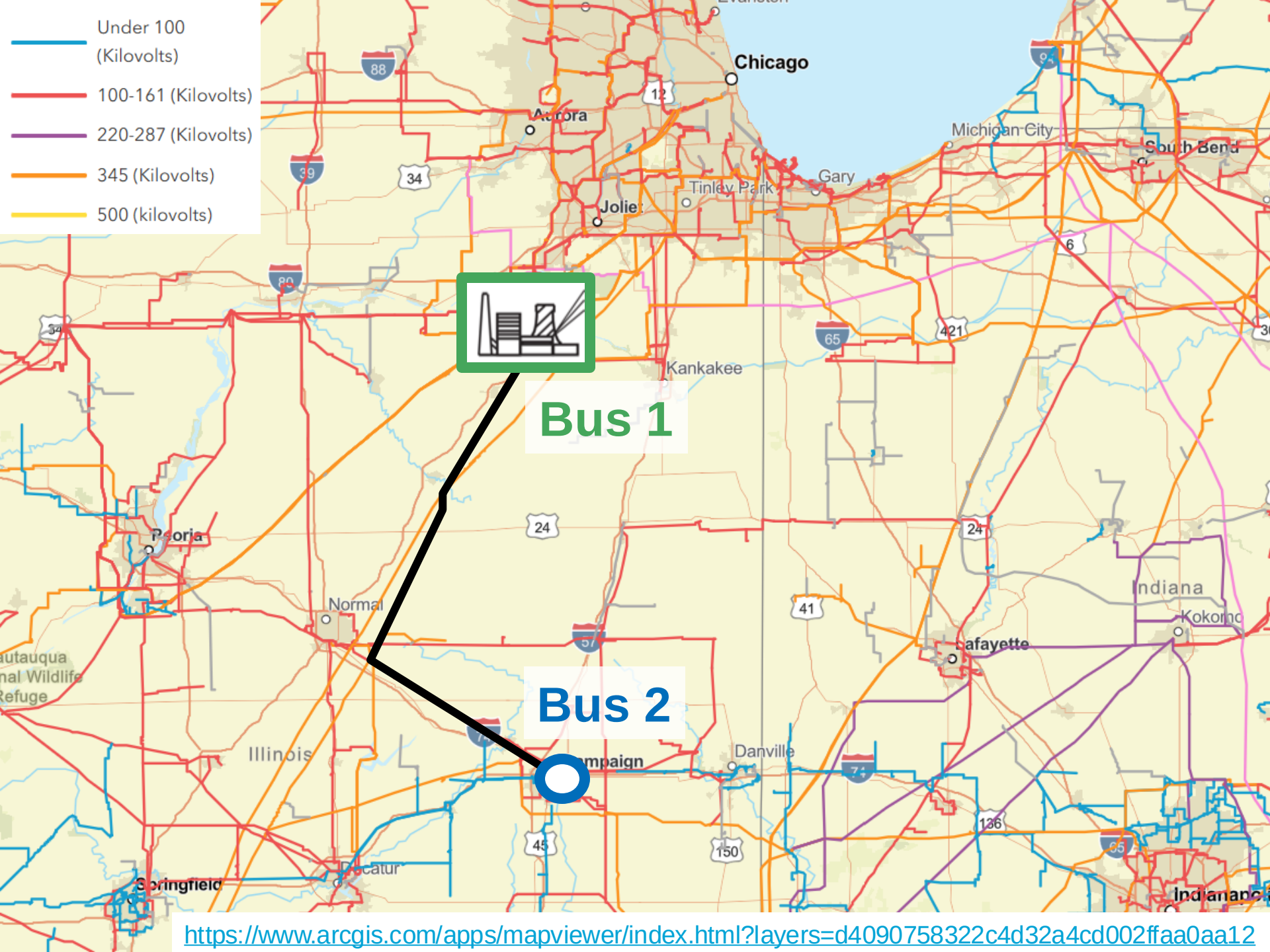
- Background: Perception on the electric grid
 - What is state estimation?
- Phase Synchronization and Eigenvectors
 - What is phase synchronization?
 - Benefit 1: Provably good estimation
 - Benefit 2: Certificate of correctness
- Future of grid perception

Power System State Estimation by Phase Synchronization and Eigenvectors

Iven Guzel and Richard Y. Zhang

arXiv:2409.12828





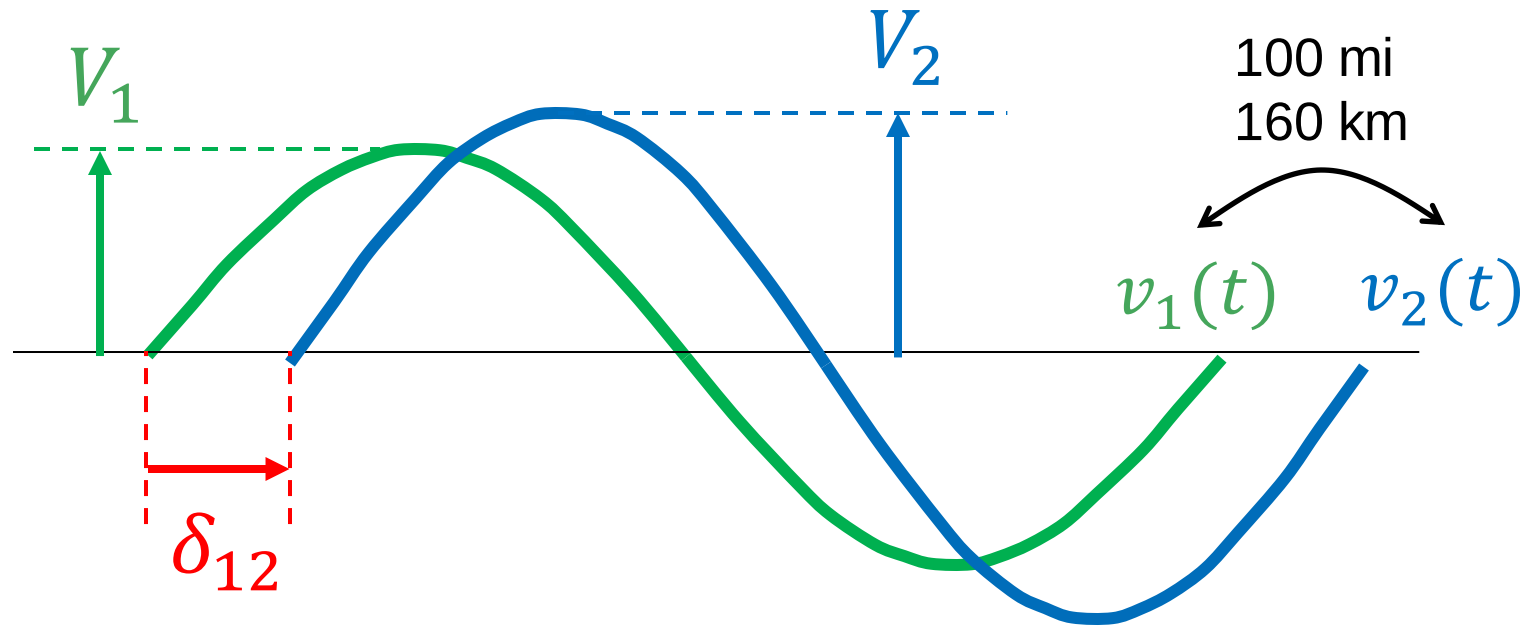
- Under 100 (Kilovolts)
- 100-161 (Kilovolts)
- 220-287 (Kilovolts)
- 345 (Kilovolts)
- 500 (kilovolts)



Bus 1

Bus 2

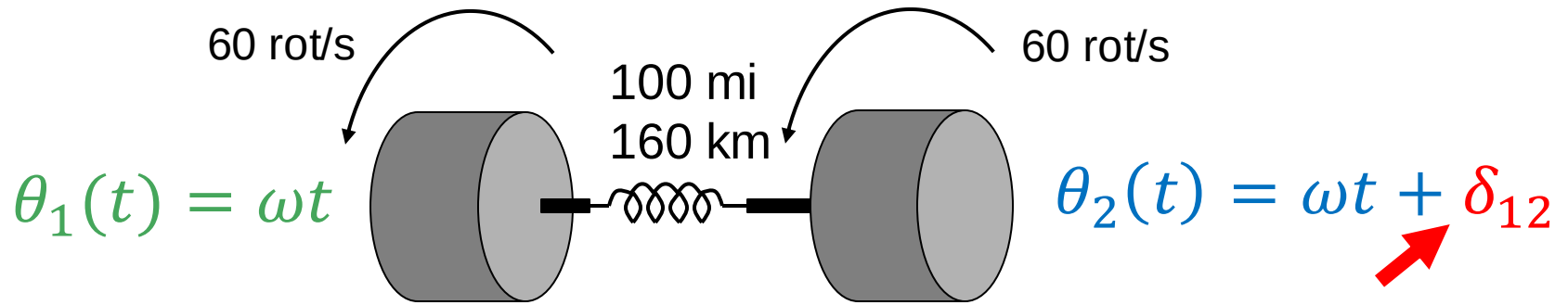
Power system state estimation



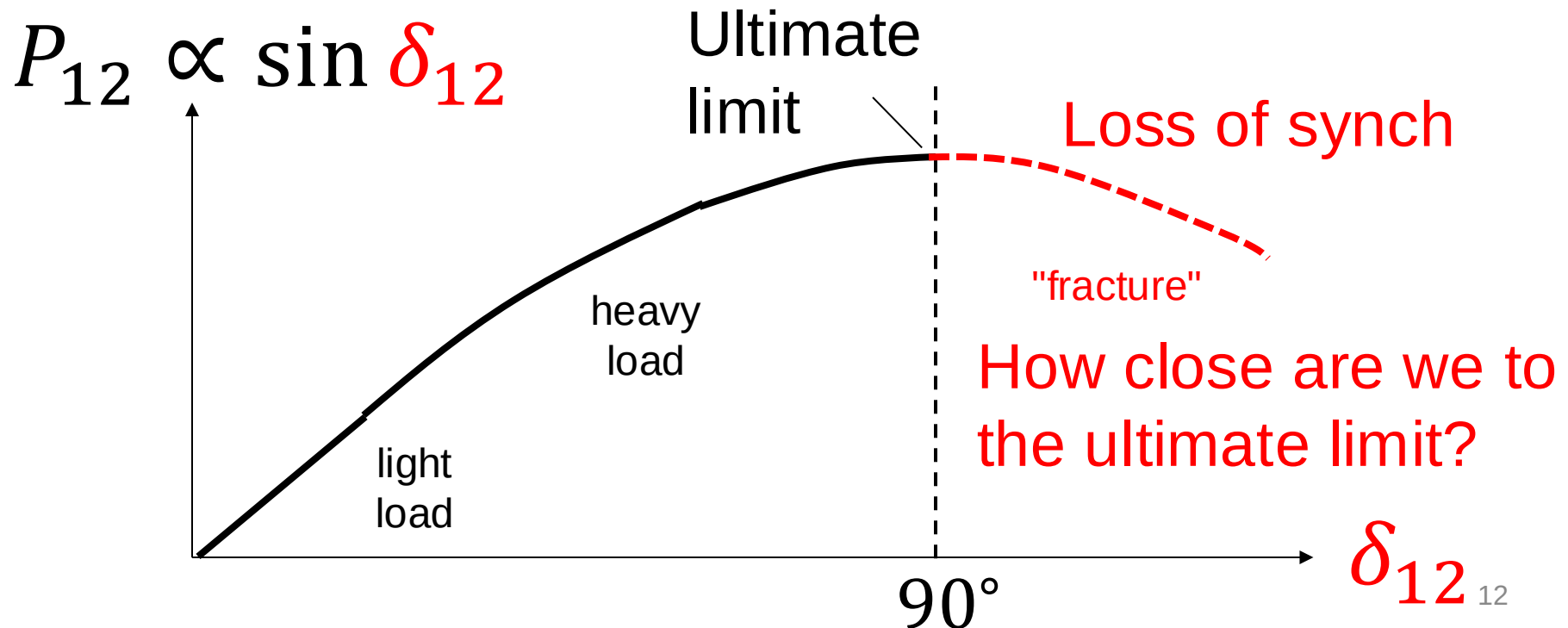
$$P_{12} = B_{12} V_1 V_2 \sin \delta_{12}$$

↑ ↑ ↑ ↑
meas known meas unknown
to be estimated

Why do we need to know the angles?



Power transfer from slight twist in the spring

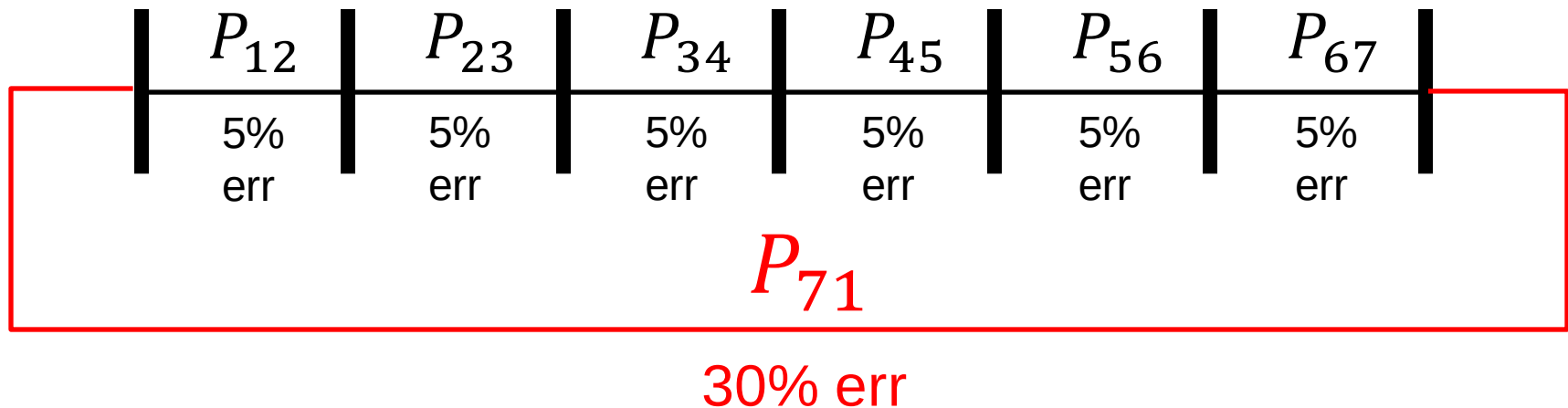


Why is estimating the angles difficult?

$$P_{12} = B_{12} V_1 V_2 \sin \delta_{12} \quad \delta_{12} = \arcsin \frac{P_{12}}{B_{12} V_1 V_2}$$

$\uparrow$ meas $\uparrow$ known $\nwarrow \nearrow$ meas $\uparrow$ unknown to be estimated

Problem: Imperfect noisy measurements



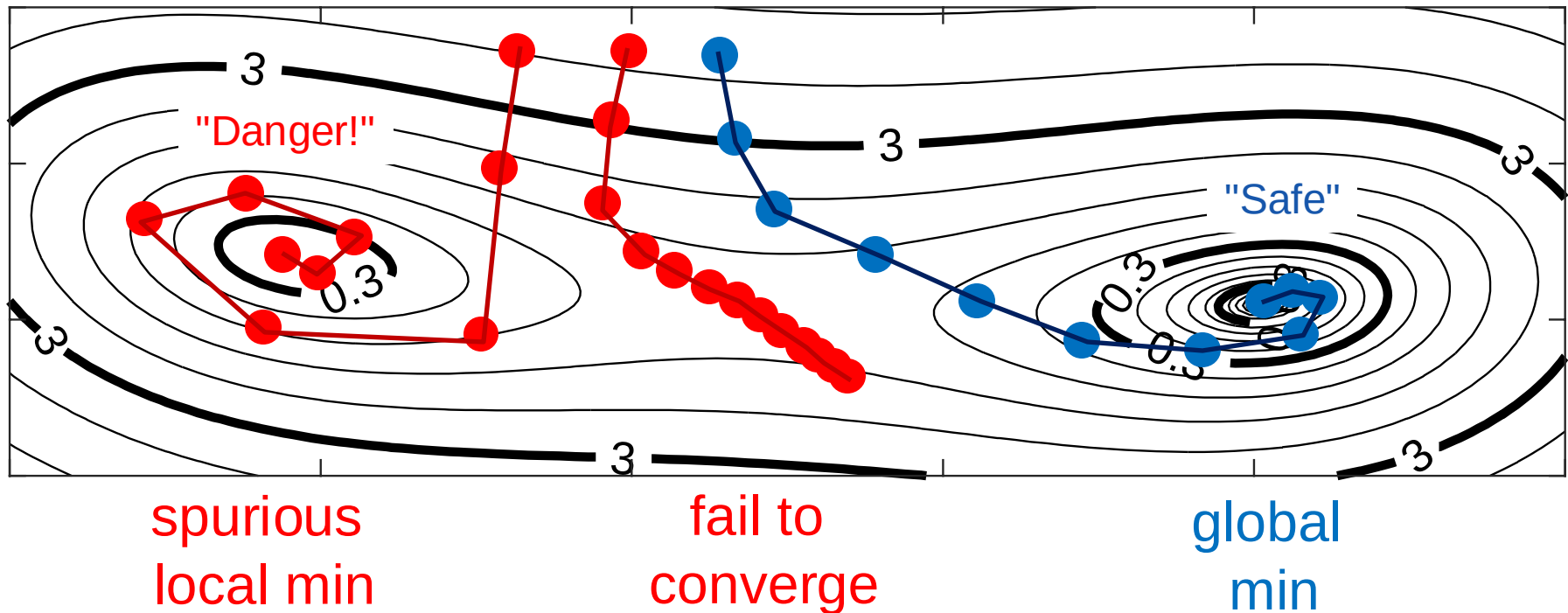
Estimation must account for all measurements

Easily ruined by a few bad measurements

Current practice and its limitations

$$\min_{\theta_1, \dots, \theta_n} \sum_{a \sim b} |P_{ab} - B_{ab} V_a V_b \sin(\theta_a - \theta_b)|^2$$

+ other power measurements



Failure mode 1: No sensible estimate

Failure mode 2: Sensible but *incorrect* estimate
(silent failure)

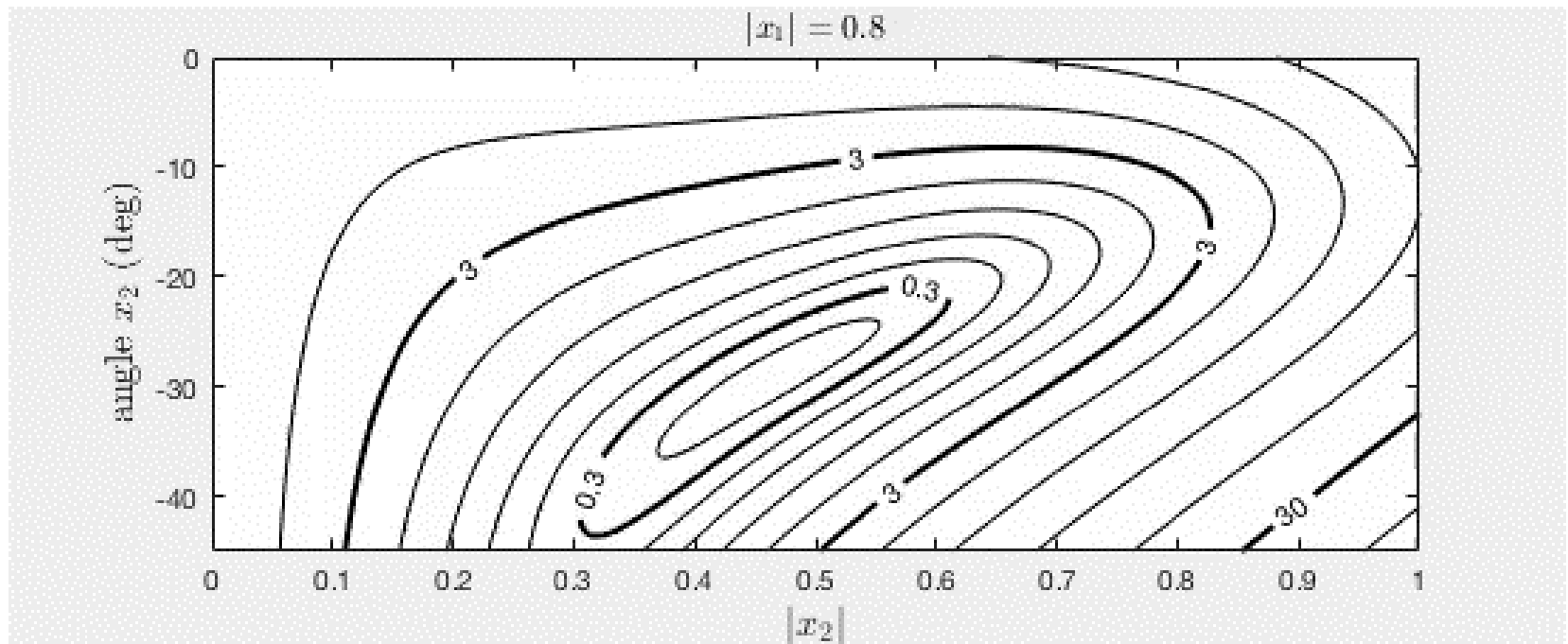
Silent failure: Plausible but incorrect

Indeed, we find four critical points, only one of which is the correct estimate

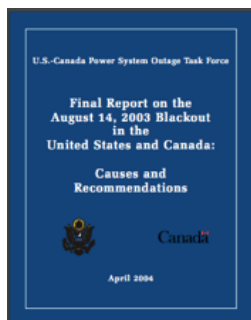
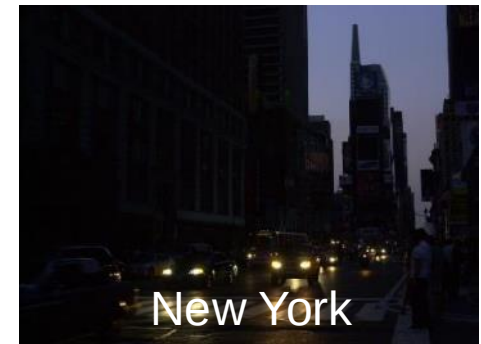
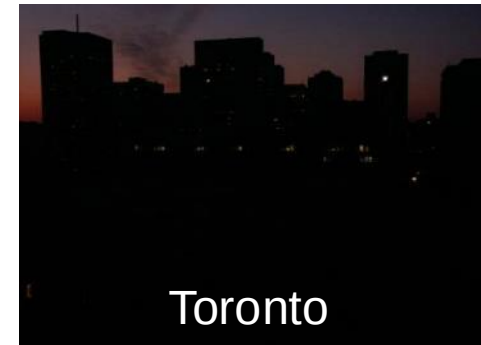
$$\begin{bmatrix} |x_1| \\ |x_2| \\ \angle x_2 \end{bmatrix} \in \left\{ \begin{bmatrix} 1 \\ 0.829 \\ -13.2^\circ \end{bmatrix}, \begin{bmatrix} 0.870 \\ 0.345 \\ -35.7^\circ \end{bmatrix}, \begin{bmatrix} 0.846 \\ 0.401 \\ -32.0^\circ \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

Global minim. Local minim. Saddle point Local maxim.

Both
estimates are
plausible!



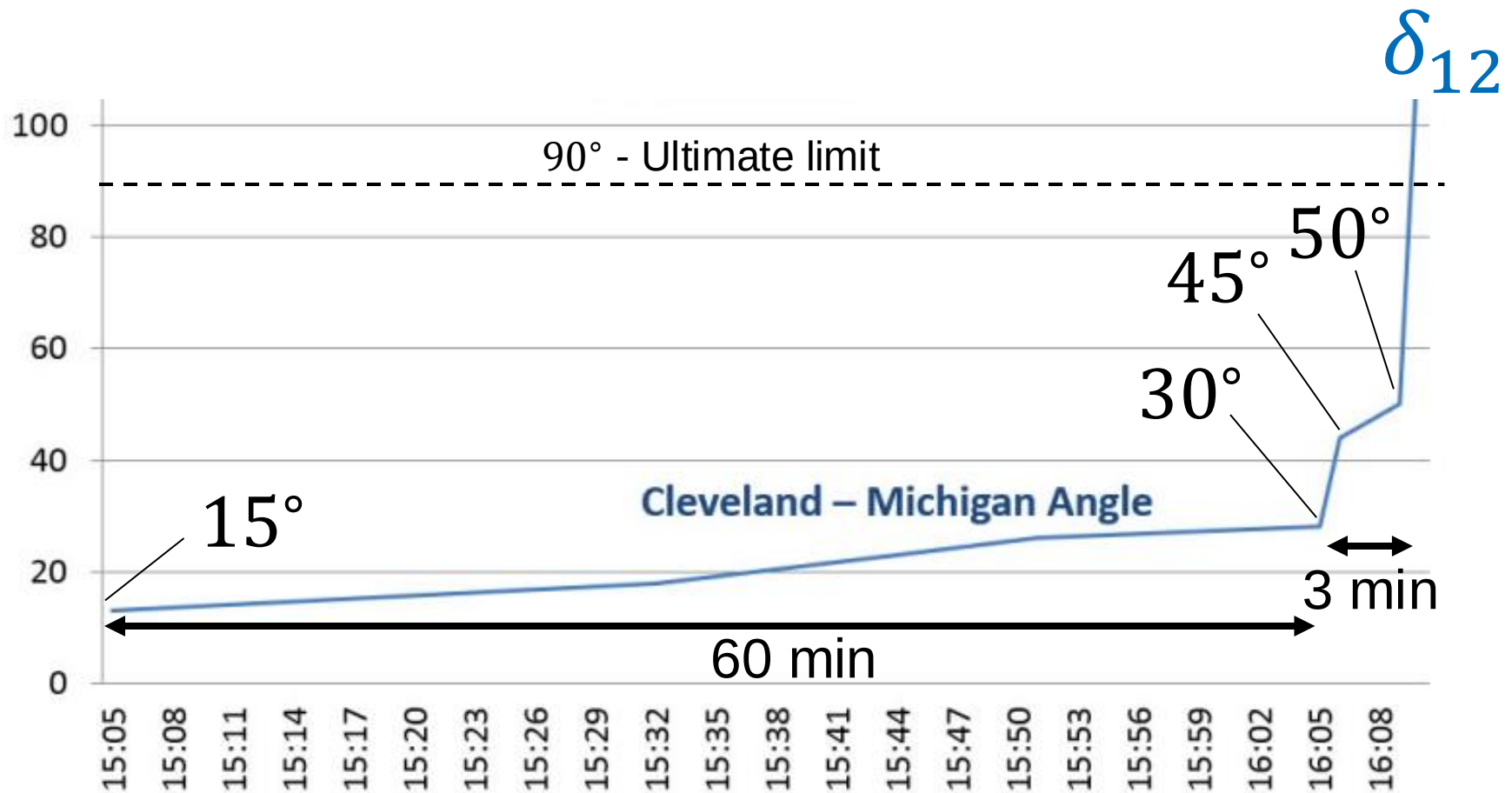
The August 14th, 2003 Northeast Blackout



“MISO’s **state estimator was faulty** for most of the period between 12:15 & 15:34 EDT.”

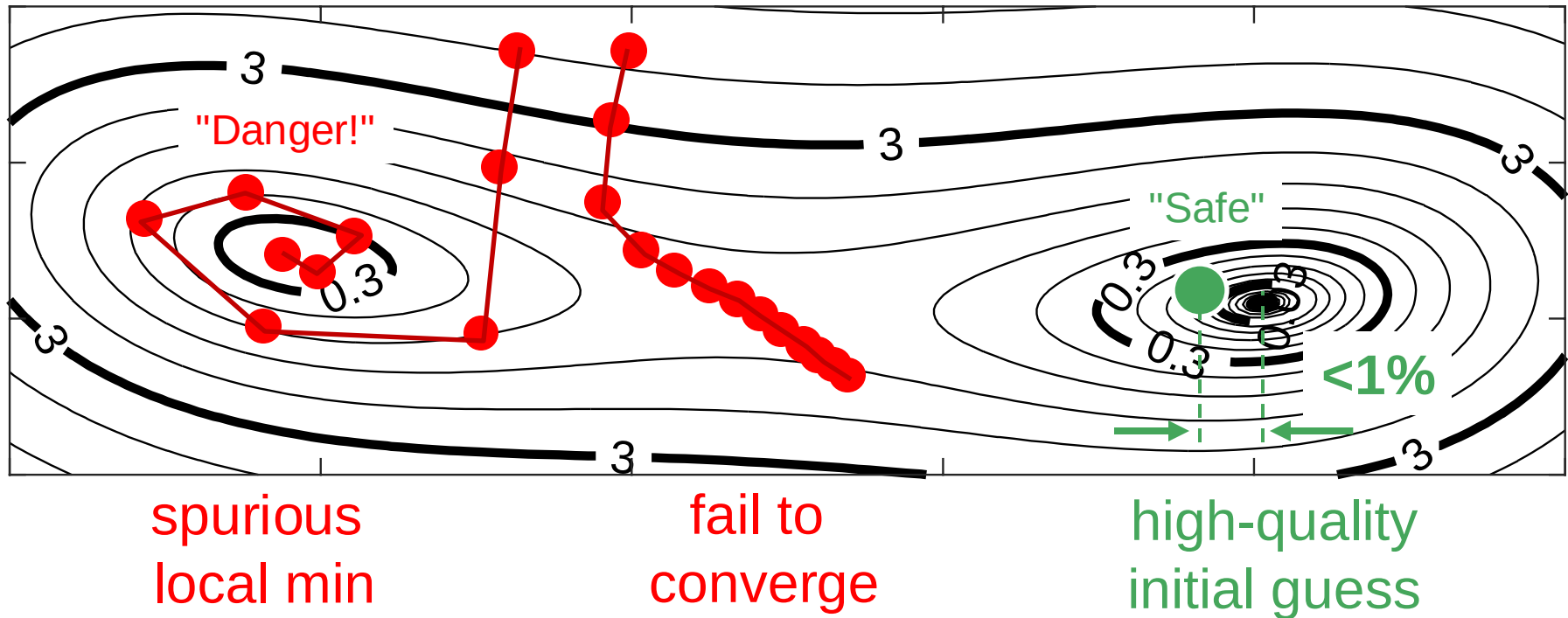
--US-Canada Power System Outage Task Force (2004)

"It can fall apart in three minutes"



Source: NERC Report, "Real-Time Application of Synchrophasors for Improving Reliability," 2010
NASPI Paper, "Using Synchrophasor Data for Phase Angle Monitoring," 2016

Today: PSSE as Phase Synchronization



Benefit 1: Provably good estimation
(from eigenvector of sparse matrix)

Benefit 2: Certificate of correctness
(from associated eigenvalue)

Usually
solve in
1 iteration

Outline

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 - What is phase synchronization?
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 - Benefit 2: Certificate of correctness
- Future of grid perception

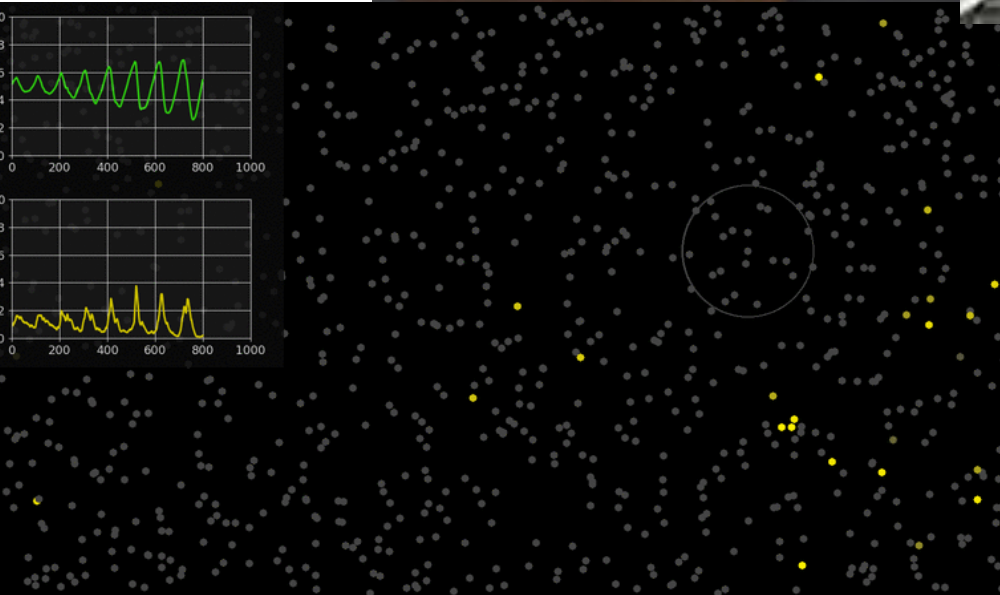
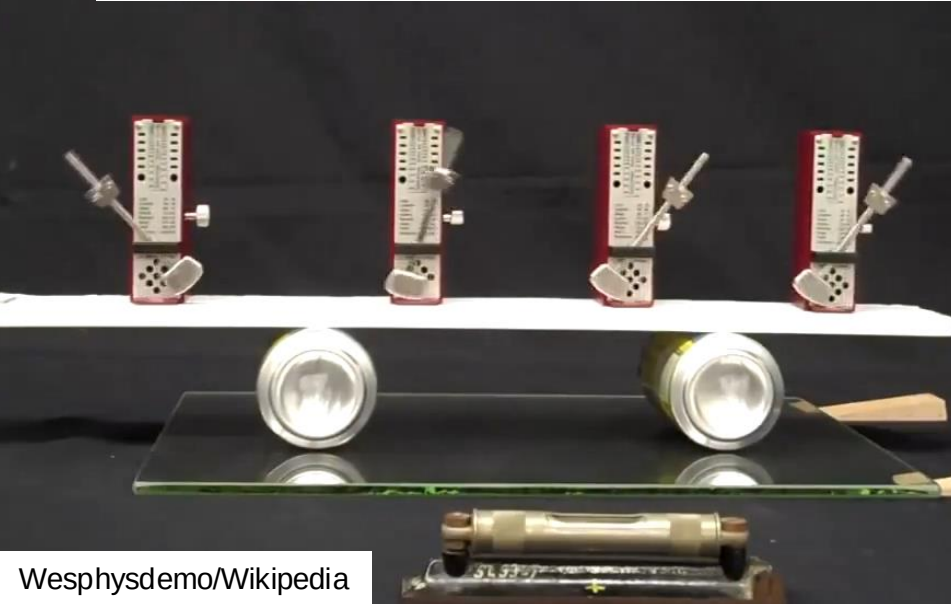
Power System State Estimation by Phase Synchronization and Eigenvectors

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arXiv:2409.12828



What is synchronization?

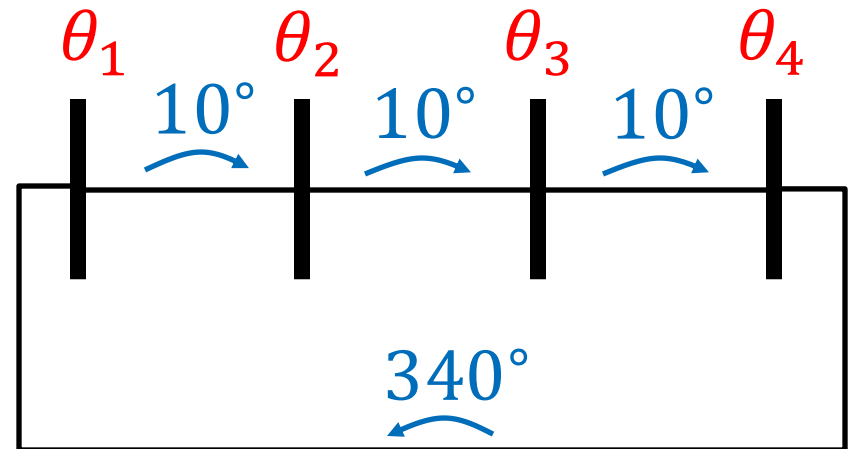
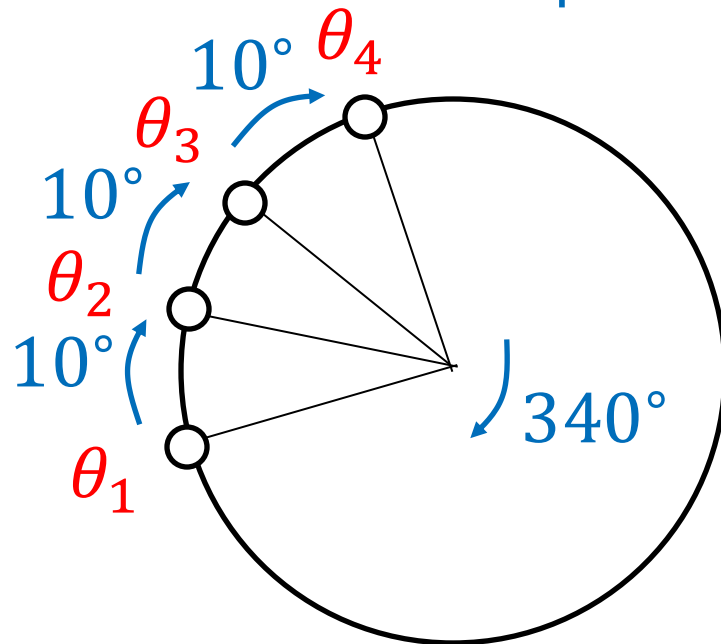


The phase synchronization problem

Under what conditions does spontaneous synchronization occur?

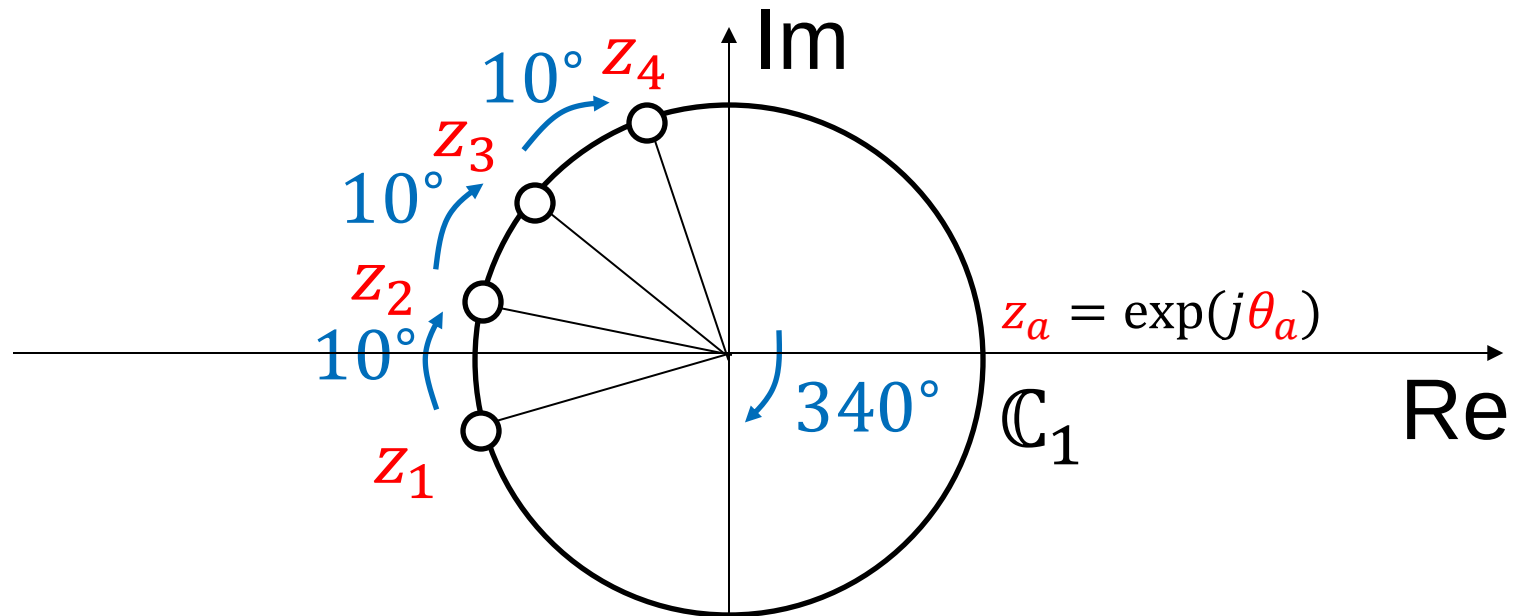
If synchronization does occur, what is the final configuration?

Find most consistent $\theta_1, \dots, \theta_n \in (-180^\circ, 180^\circ]$
such that $\theta_a - \theta_b \approx \delta_{a,b} \pmod{360^\circ}$ for all $a \sim b$
pairwise difference measured pairs



Phase sync as nonconvex optimization

Find most consistent $z_1, \dots, z_n \in \mathbb{C}_1$
such that $z_a \approx z_b \exp(j\delta_{a,b})$ for all $a \sim b$



$$\min_{\substack{z_1, \dots, z_n \in \mathbb{C} \\ |z_a|^2 = 1}} \sum_{a \sim b} |z_a - z_b \exp(j\delta_{a,b})|^2$$

Minimum energy configuration

Eigenvector method [Singer 2011]

$$\min_{\substack{\mathbf{z}_1, \dots, \mathbf{z}_n \in \mathbb{C} \\ |\mathbf{z}_a|^2 = 1}} \sum_{a \sim b} |\mathbf{z}_a - \mathbf{z}_b \exp(j\delta_{a,b})|^2$$


spectral relaxation (also certifies correctness)

$$\geq \min_{\substack{\mathbf{z}_1, \dots, \mathbf{z}_n \in \mathbb{C} \\ \sum_a |\mathbf{z}_a|^2 = n}} \sum_{a \sim b} |\mathbf{z}_a - \mathbf{z}_b \exp(j\delta_{a,b})|^2$$

$$= \min_{\substack{\mathbf{z} \in \mathbb{C}^n \\ \|\mathbf{z}\|^2 = n}} \|F\mathbf{z}\|^2 = n\lambda_{\min}(F^*F)$$

optimal $\mathbf{z}$ is bottom
eigenvector of F^*F



Eigenvector method [Singer 2011]

Find most consistent $z_1, \dots, z_n \in \mathbb{C}_1$
such that $z_a \approx z_b \exp(j\delta_{a,b})$ for all $a \sim b$

1. Compute optimal $z_1, \dots, z_n$ as sparse eigenvector problem

$$\geq \min_{\substack{z_1, \dots, z_n \in \mathbb{C} \\ \sum_a |z_a|^2 = n}} \sum_{a \sim b} |z_a - z_b \exp(j\delta_{a,b})|^2$$

2. Project back onto the complex circle $\mathbb{C}_1$

$$z_a \leftarrow z_a / |z_a|$$

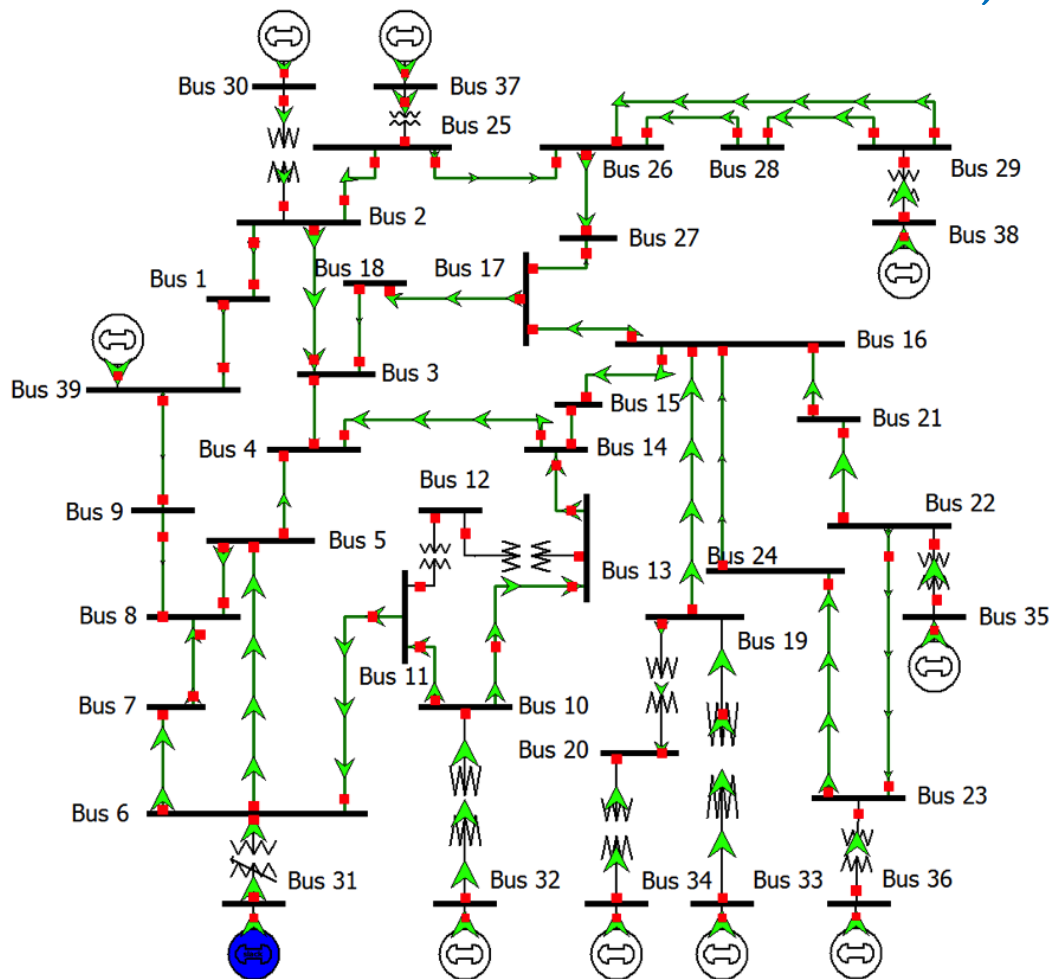
3. Use eigenvalue lower-bound to certify correctness

If $\delta_{a,b}$ near-consistent and connected, then one-shot near-recovery

Theorem [Filbir, Krahmer, Melnyk]. Estimation error scales linearly with noise. Leading constant is inversely proportional to the algebraic connectivity of the graph.

Given all magnitudes, find all angles?

Find most consistent $v_1, \dots, v_n \in \mathbb{C}, |v_a|^2 = 1$ $\nwarrow$
such that $v_a \approx v_b \exp(j\delta_{a,b})$ for all $a \sim b$ without loss of generality



1. Convert all branch power meas S_{ab} into angle differences δ_{ab}
2. Solve for $v_a = \exp(j\theta_a)$

Critical issue:
What about bus power meas S_a ?

Proposed method [Guzel & Zhang 2024]

$$\min_{\substack{\mathbf{v}_1, \dots, \mathbf{v}_n \in \mathbb{C} \\ |\mathbf{v}_a|^2 = 1}} \sum_a |\mathbf{v}_a i_a^* - \mathbf{S}_a|^2 \quad \text{for } i = Y_{bus} \mathbf{v} \quad \leftarrow$$

critical insight: exact reformulation

$$= \min_{\substack{\mathbf{v}_1, \dots, \mathbf{v}_n \in \mathbb{C} \\ |\mathbf{v}_a|^2 = 1}} \|[Y_{bus} - \text{diag}(\mathbf{S}^*)] \mathbf{v}\|^2$$

without loss of generality

spectral relaxation (certifies correctness)

$$\geq \min_{\substack{\mathbf{v} \in \mathbb{C}^n \\ \|\mathbf{v}\|^2 = n}} \|[Y_{bus} - \text{diag}(\mathbf{S}^*)] \mathbf{v}\|^2$$

$$= \min_{\substack{\mathbf{v} \in \mathbb{C}^n \\ \|\mathbf{v}\|^2 = n}} \|C \mathbf{v}\|^2 = n \lambda_{\min}(C^* C)$$

optimal $\mathbf{v}$ is bottom eigenvector of $C^* C$

Proposed method [Guzel & Zhang 2024]

Find most consistent $\mathbf{v}_1, \dots, \mathbf{v}_n \in \mathbb{C}, |\mathbf{v}_a|^2 = 1$
such that $\mathbf{v}_a i_a^* \approx \mathbf{S}_a$ for $i = Y_{bus} \mathbf{v}$ for all a

1. Compute optimal $\mathbf{v}_1, \dots, \mathbf{v}_n$ as sparse eigenvector problem

$$\geq \min_{\substack{\mathbf{v} \in \mathbb{C}^n \\ \|\mathbf{v}\|^2 = n}} \|[Y_{bus} - \text{diag}(\mathbf{S}^*)]\mathbf{v}\|^2$$

2. Assign the correct (measured) voltage magnitudes

$$\mathbf{v}_a \leftarrow \mathbf{v}_a / |\mathbf{v}_a|$$

3. Use eigenvalue lower-bound to certify correctness

If $\mathbf{S}_a$ near-consistent and connected, then one-shot near-recovery

Theorem [McRae & Zhang]. Estimation error scales linearly with noise. Leading constant is inversely proportional to the algebraic connectivity of the graph.

(For full generalization to mixed branch / bus measurements, PMU measurements, missing voltage magnitudes, see Guzel & Zhang)

European System Model (PEGASE)

buses

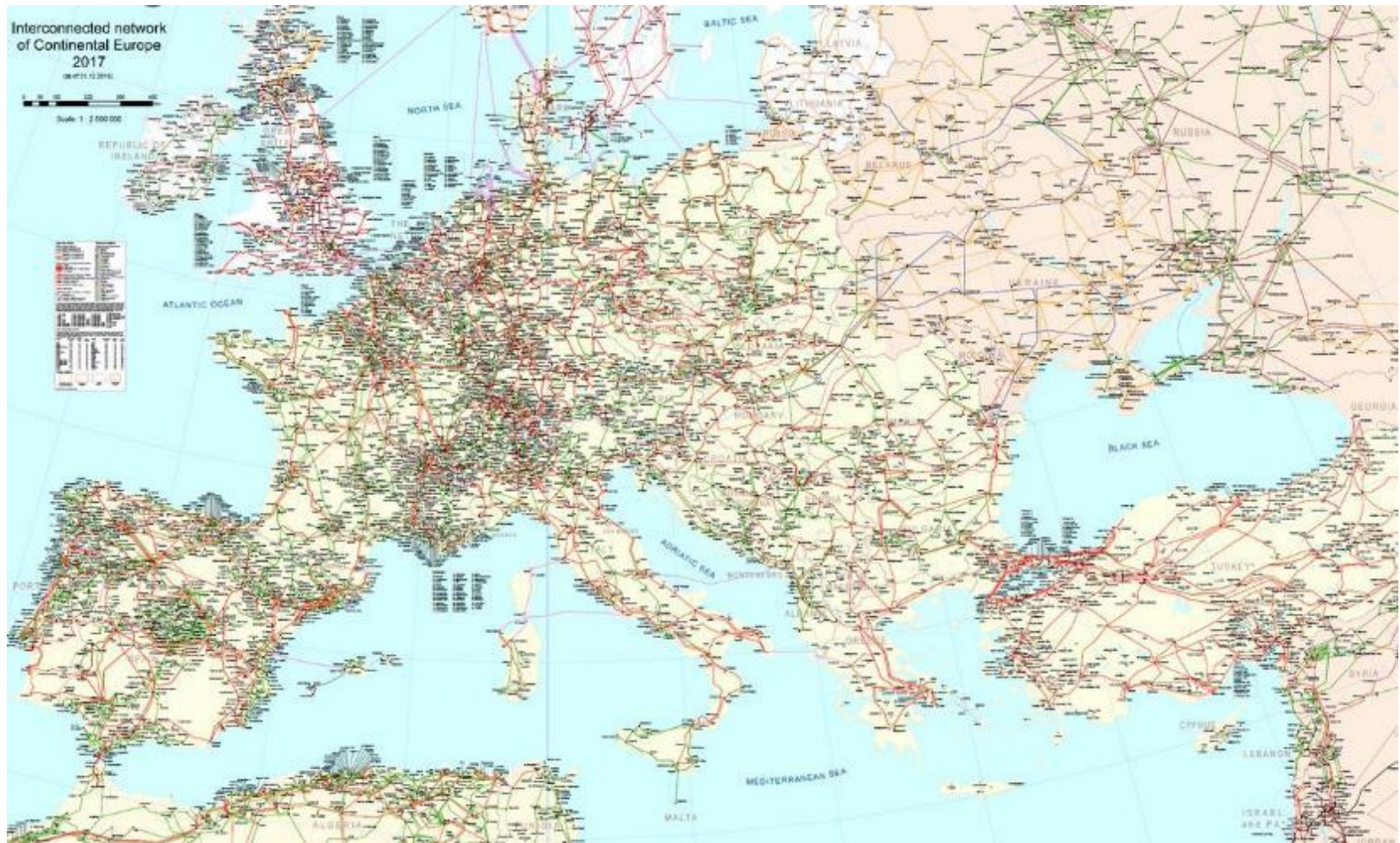
$n = 13,659$

branches

$|E| = 20,467$

generators

$|G| = 4,092$



Source: Cédric Josz, Stéphane Fliscounakis, Jean Maeght, Patrick Panciatici (2016)

Error scales linearly with noise

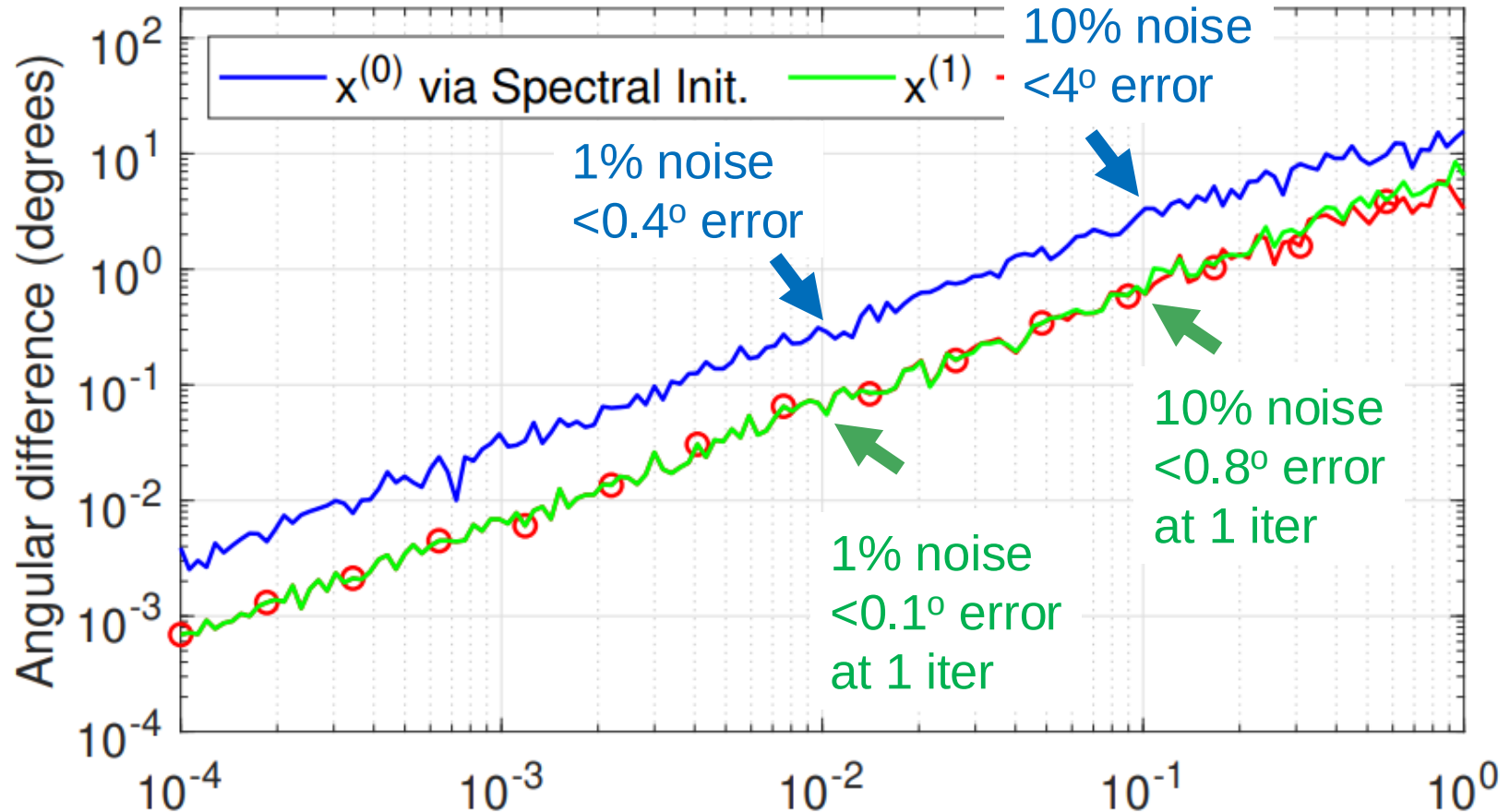
buses

$n = 13,659$

meas

$m_{\text{bus}} = 27,318$

← Only PQ bus meas



noise absolute standard deviation (per unit)

>99.9% certified optimality with 1 iter

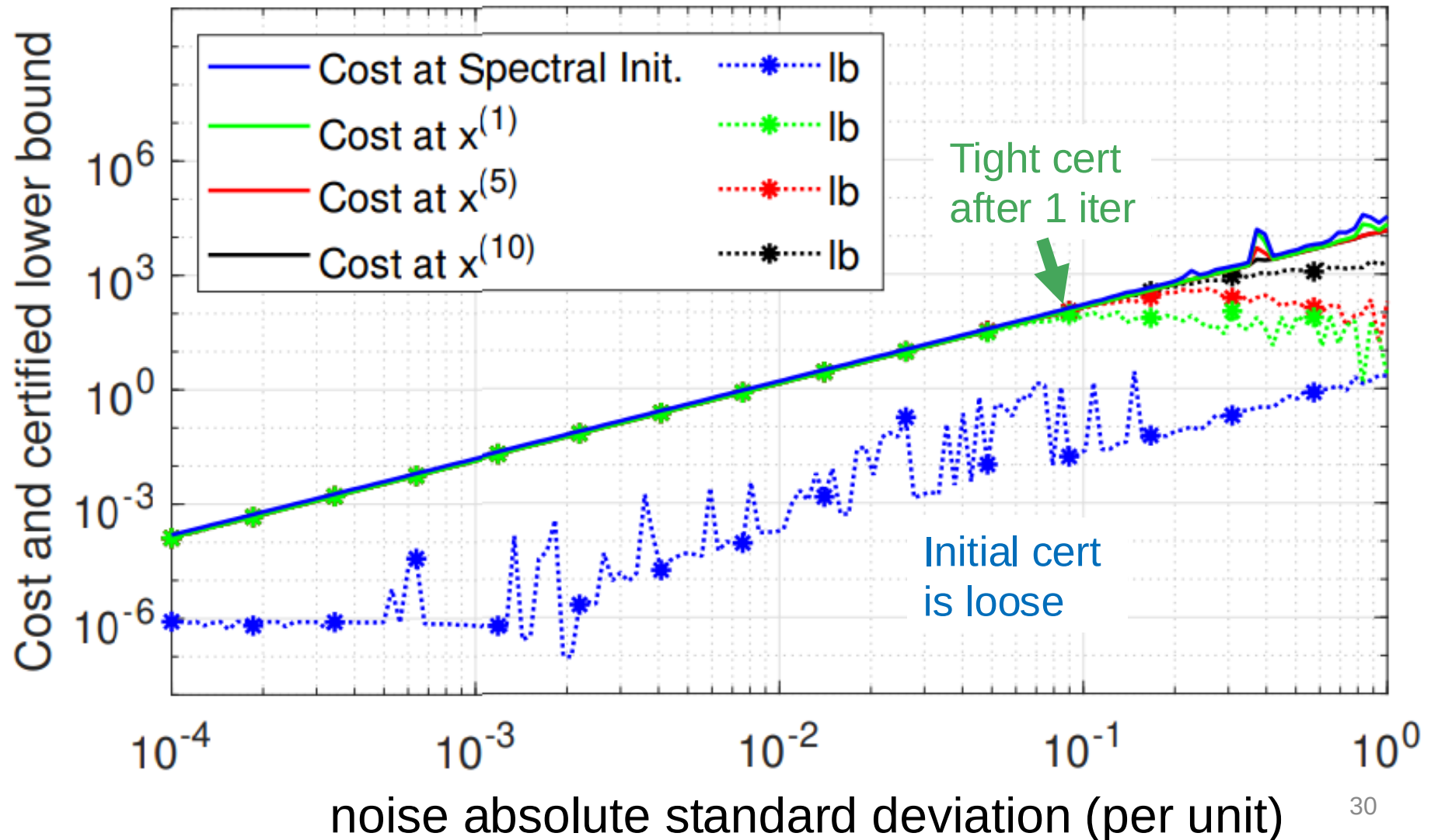
buses

$n = 13,659$

meas

$m_{\text{bus}} = 27,318$

← Only PQ bus meas



Running time on laptop (sec)

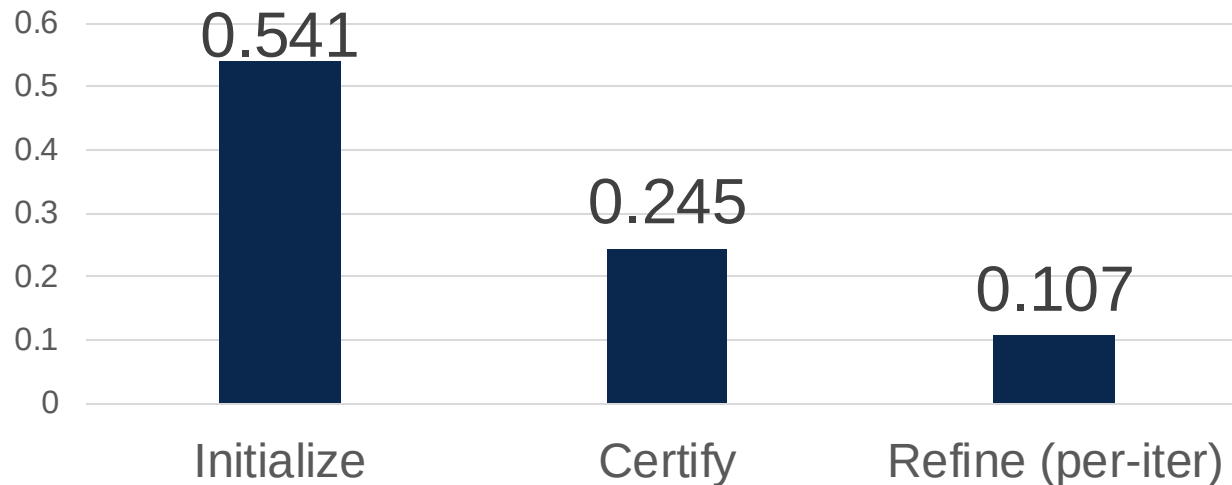
buses

$n = 13,659$

meas

$m_{\max} = 109,186$

← All PQ branch
+ PQ bus meas



Benefit 1: Provably good estimation
(from eigenvector of sparse matrix)

$< 1/2$ sec

Benefit 2: Certificate of correctness
(from associated eigenvalue)

$< 1/4$ sec

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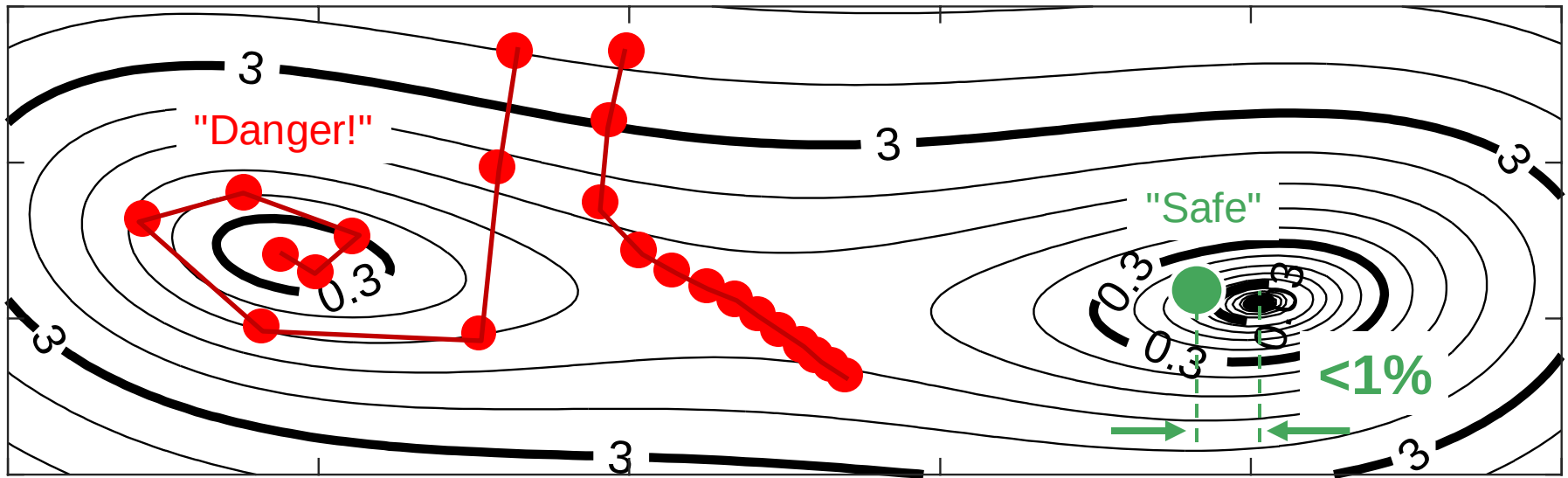
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Remaining questions on Phase Sync



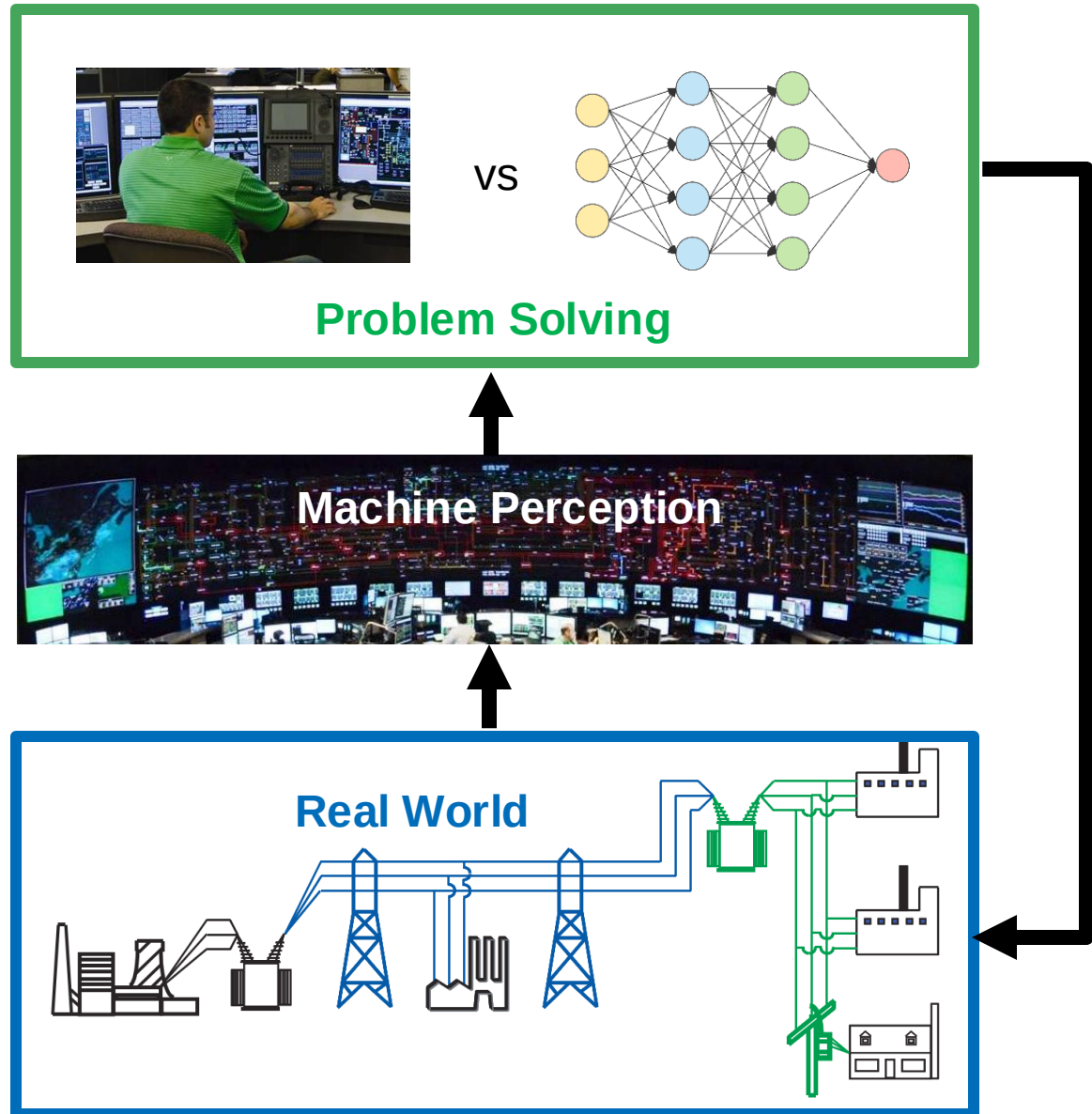
Is the method resilient to bad measurements?

Initial experiments: Yes, another key benefit!

“We successfully estimate $n = 400$ angles from a full set of $m = \binom{400}{2}$ offset measurements of which 90% are outliers” – Singer [2011]

Can it resist *adversarial* bad measurements?

Remaining gaps on grid perception



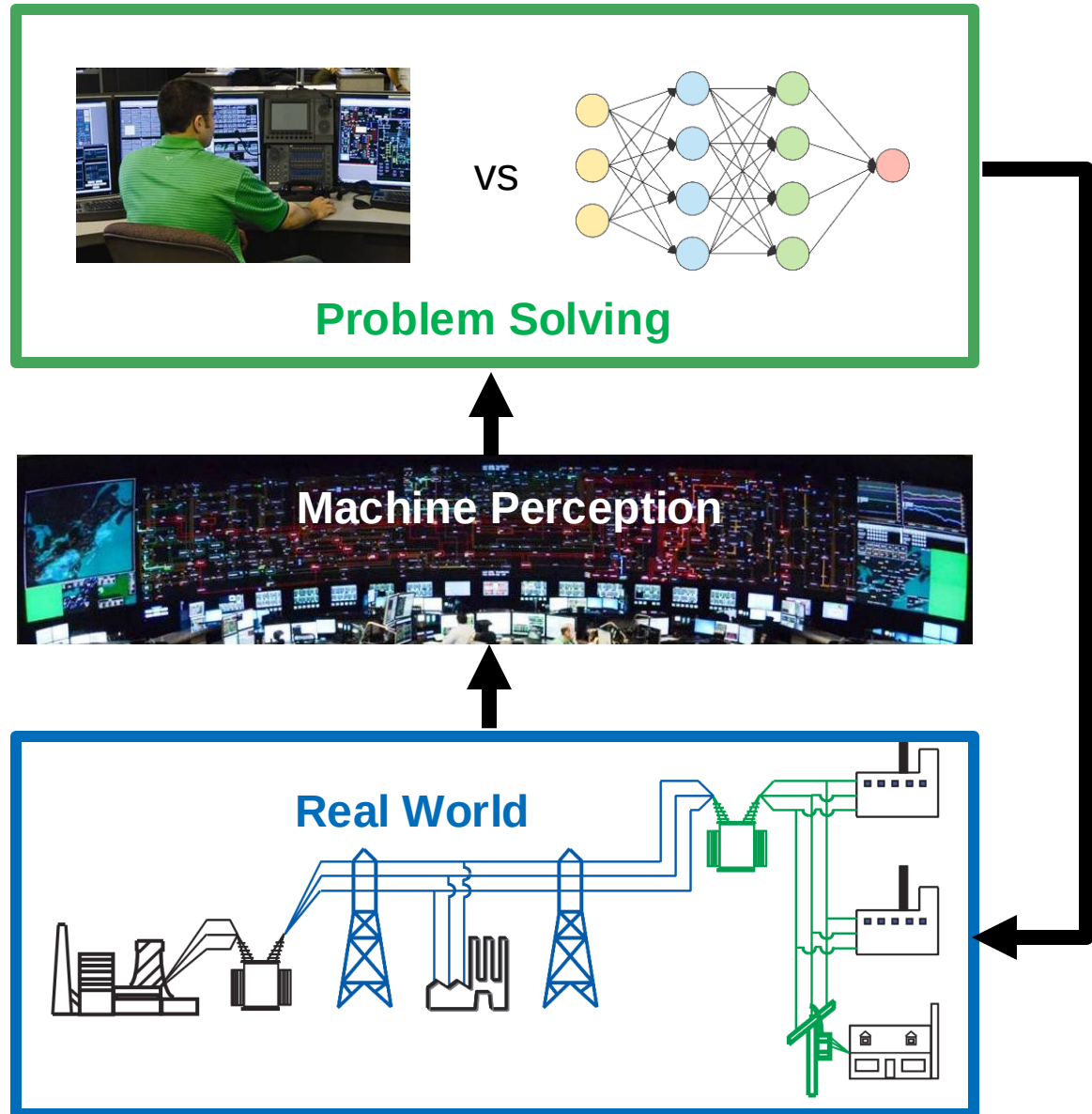
Perception

- Sensing
- Estimation
- Inference
- Semantic understanding

Extend Phase Sync to grid operations?

Problem Solving

- Dispatch
- Scheduling
- Planning



Conclusions - Thank you!

- AI for cyber-physical systems: Perception is the critical bottleneck.
- Current approach to grid perception susceptible to failure; silent failure particularly alarming.
- This paper: Fundamental connection with Phase Synchronization. Provably good initialization, certification of correctness.
- Future: Resilience; Semantic understanding; Application to operations?

Email: ryz@illinois.edu

