

Measurement-based Estimation and Control of Power Distribution Systems: Theory and Experimental Validation

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Outline

- Motivation
- Network Parameter Estimation Problem
- Application: Voltage Control using estimated parameters
 - Validation on real-microgrid
- State Estimation under Limited Observability

Review paper on Power System
Estimation Techniques:

Learning Power Flow Models and Constraints From Time-Synchronized Measurements: A Review

This article presents a comprehensive review of measurement-based estimation methods for power flow models using time-synchronized measurements.

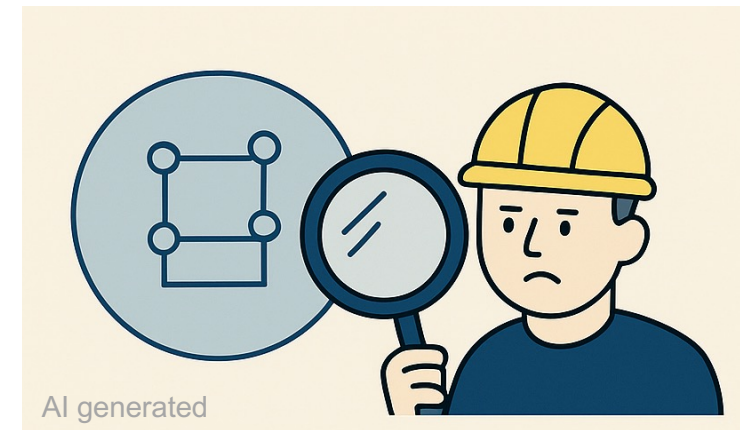
By RAHUL K. GUPTA[✉], Member IEEE, PAOLO ATTILIO PEGORARO[✉], Senior Member IEEE,
OGNEN STANOJEV, Student Member IEEE, ALI ABUR[✉], Life Fellow IEEE,
CARLO MUSCAS[✉], Senior Member IEEE, GABRIELA HUG[✉],
AND MARIO PAOLONE[✉] Fellow IEEE



Motivation: “Invisible” Power System

Why do we need estimation in power system? What are the common causes for the invisibility or unavailability of power network models?

- Limited or no prior information about the network topology parameters.
- Inaccurate or outdated database
- Infrastructure ageing is leading to a change in the parameters
- Impact of weather (change in temperature, etc.)
- Network control (topology reconfiguration)



Offline and Online Estimation Schemes

Offline:

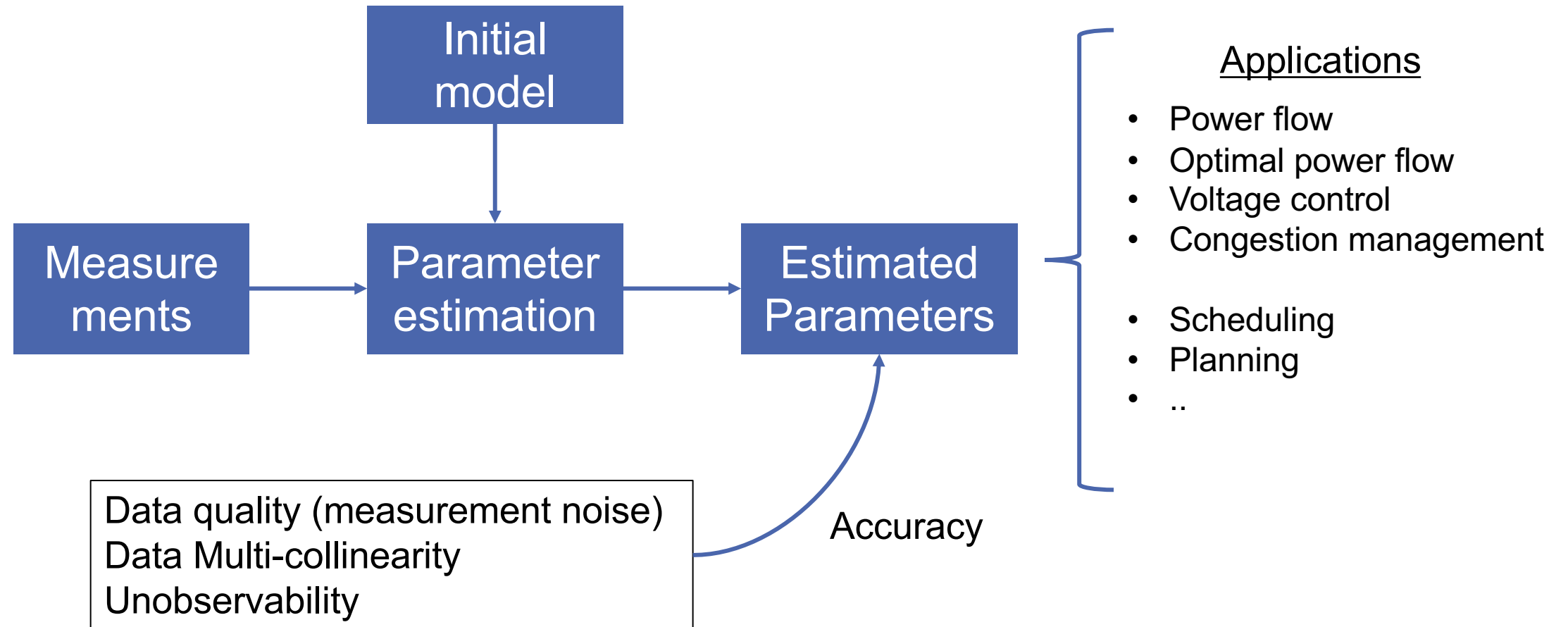
- The network **branch and shunt parameters** which do not change.
- These parameters have been verified to work well for operation and planning schemes.
- It can leverage a large amount of historical data.

Online:

- Estimating **states** (Voltage magnitudes and angles)
- Network parameter that change with such as due to **ageing, temperature variations**, shunt/reactance switching, topology change etc.
- Parameter **correction**.
- It can be also used for estimating sensitivity coefficients (depends on injections).
- Requires online (recent) measurements.



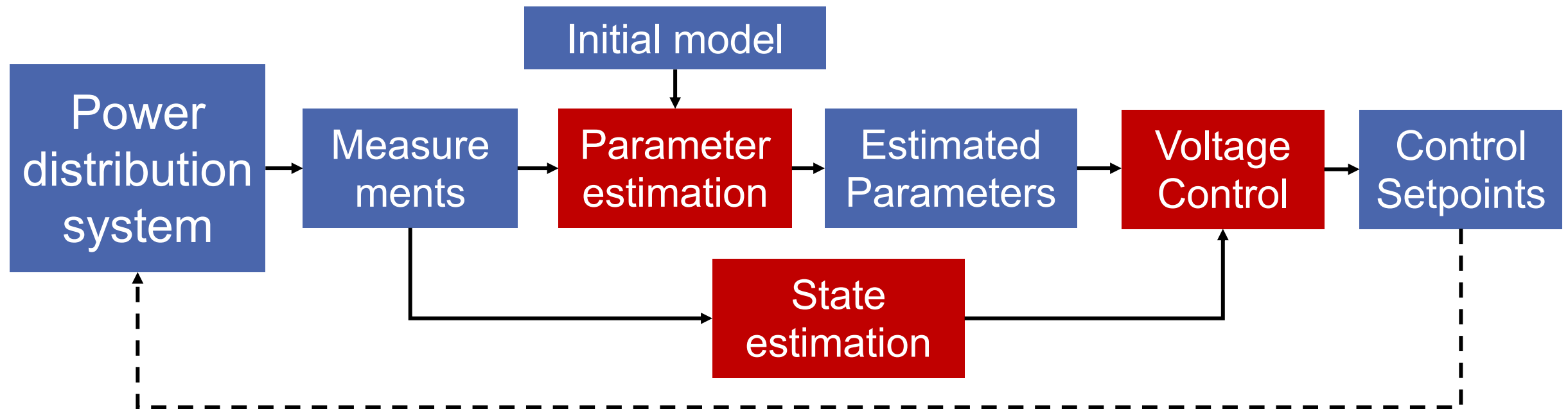
Estimation and Application Framework



This talk: Measurement-based Control on “Invisible” or Partially known Distribution System

Consider a distribution network whose network parameters are not accurately known.

Objective: to develop a **measurement-based** and **real-time** framework for **controlling distributed energy resources** in distribution systems (e.g., for voltage control). **Experimentally validate** on a real operational network.



Power System Parameter Estimation

- **State-independent models:**

- Estimation of branch and shunt parameters (those don't depend on the state of the network)
- Topology estimation

- **State-dependent models:**

- Estimation of sensitivity coefficients, Jacobian etc. distribution factors

Gupta, R. K., Pegoraro, P. A., Stanojev, O., Abur, A., Muscas, C., Hug, G., & Paolone, M. (2025). Learning Power Flow Models and Constraints From Time-Synchronized Measurements: A Review. *Proceedings of the IEEE*



State-independent models: Branch and Shunt Parameter Estimation Problem

Objective: estimate the branch and shunt parameters

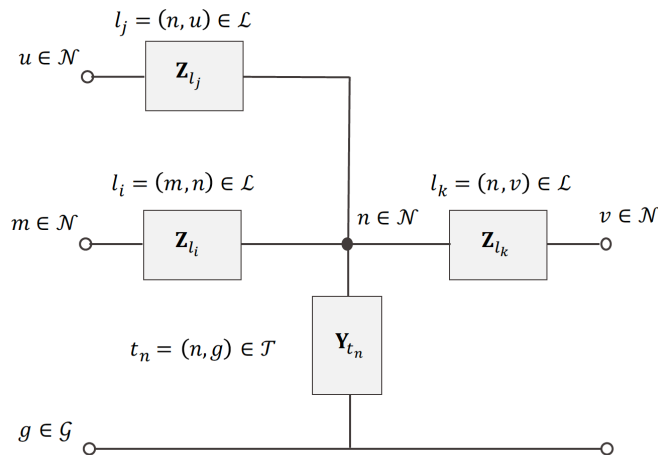
Assumptions:

- Topology known
- Measurement from all the nodes (offline)



Estimation model – Balanced and Transposed

Nodal voltages and injection currents measurements



Estimation model

$$\mathbf{v} := \text{col}_{n \in \mathcal{N}}(\mathbf{v}_n)$$

$$\mathbf{i} := \text{col}_{n \in \mathcal{N}}(\mathbf{i}_n)$$

$$\mathbf{Y} = (\mathcal{A}^{\mathcal{P}})^T \mathbf{Y}_{\mathcal{L}} \mathcal{A}^{\mathcal{P}} + \mathbf{Y}_{\mathcal{T}}$$

$$\mathbf{Y}_{\mathcal{L}} = \text{diag}_{l \in \mathcal{L}}(\mathbf{Y}_l)$$

$$\mathbf{Y}_{\mathcal{T}} = \text{diag}_{t \in \mathcal{T}}(\mathbf{Y}_t)$$

Admittance matrix and primitives

$\mathcal{A}^{\mathcal{P}}$ is the polyphase incidence matrix

Kirchhoff's laws

$$\mathbf{i} = ((\mathcal{A}^{\mathcal{P}})^T \mathbf{Y}_{\mathcal{L}} \mathcal{A}^{\mathcal{P}} + \mathbf{Y}_{\mathcal{T}}) \mathbf{v}$$

$$\mathbf{i} = (\mathcal{A}^{\mathcal{P}})^T \mathbf{i}_{\mathcal{L}} + \mathbf{i}_{\mathcal{T}}$$

$$\mathbf{i} = \mathcal{A}^T \text{diag}(\mathcal{A} \mathbf{v}) \text{col}_{l \in \mathcal{L}}(\mathbf{Y}_l) + \text{diag}_{n \in \mathcal{N}}(\mathbf{v}_n) \text{col}_{t \in \mathcal{T}}(\mathbf{Y}_t)$$

$$\mathbf{i}_{\mathcal{L}} = \text{diag}(\mathcal{A} \mathbf{v}) \text{col}_{l \in \mathcal{L}}(\mathbf{Y}_l)$$

$$\begin{bmatrix} \Re\{\mathbf{i}\} \\ \Im\{\mathbf{i}\} \end{bmatrix} = \begin{bmatrix} \mathcal{A}^T \mathbf{C} & -\mathcal{A}^T \mathbf{D} & -\mathbf{E} \\ \mathcal{A}^T \mathbf{D} & \mathcal{A}^T \mathbf{C} & \mathbf{F} \end{bmatrix} \begin{bmatrix} \mathbf{X}^G \\ \mathbf{X}^B \\ \mathbf{X}^T \end{bmatrix},$$

$$\begin{bmatrix} \Re\{\mathbf{i}_{\mathcal{L}}\} \\ \Im\{\mathbf{i}_{\mathcal{L}}\} \end{bmatrix} = \begin{bmatrix} \mathbf{C} & -\mathbf{D} \\ \mathbf{D} & \mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{X}^G \\ \mathbf{X}^B \end{bmatrix}$$

$$\mathbf{C} = \text{diag}(\mathcal{A} \Re\{\mathbf{v}\})$$

$$\mathbf{D} = \text{diag}(\mathcal{A} \Im\{\mathbf{v}\})$$

$$\mathbf{E} = \text{diag}(\Im\{\mathbf{v}\})$$

$$\mathbf{F} = \text{diag}(\Re\{\mathbf{v}\})$$

$$\mathbf{X}^G = \text{col}_{l \in \mathcal{L}}(\Re\{\mathbf{Y}_l\})$$

$$\mathbf{X}^B = \text{col}_{l \in \mathcal{L}}(\Im\{\mathbf{Y}_l\})$$

$$\mathbf{X}^T = \text{col}_{t \in \mathcal{T}}(\Im\{\mathbf{Y}_t\})$$

R.K. Gupta, F. Sossan, J.-Y., Le Boudec, and M. Paolone, "Compound admittance matrix estimation of three-phase untransposed power distribution grids using synchrophasor measurements", IEEE Transactions on Instrumentation and Measurement, 70, pp.1-13. 2021.



Estimation model – Three-phase Untransposed

Transformation to express 3-ph impedance matrix estimates into a vector form

$$\Xi(\mathbf{Y}_l) = \Xi \left(\begin{bmatrix} Y_l^{11} & Y_l^{12} & Y_l^{13} \\ Y_l^{12} & Y_l^{22} & Y_l^{23} \\ Y_l^{13} & Y_l^{23} & Y_l^{33} \end{bmatrix} \right) = \begin{bmatrix} Y_l^{11} \\ Y_l^{22} \\ Y_l^{33} \\ Y_l^{12} \\ Y_l^{23} \\ Y_l^{13} \end{bmatrix}$$

$$f_Y(\mathbf{v}_n) = f_Y \left(\begin{bmatrix} v_{n,1} \\ v_{n,2} \\ v_{n,3} \end{bmatrix} \right) = \begin{bmatrix} v_{n,1} & 0 & 0 & v_{n,2} & 0 & v_{n,3} \\ 0 & v_{n,2} & 0 & v_{n,1} & v_{n,3} & 0 \\ 0 & 0 & v_{n,3} & 0 & v_{n,2} & v_{n,1} \end{bmatrix}$$

$$\Lambda(\mathbf{v}) = \text{col}_{n \in \mathcal{N}}(f_Y(\mathbf{v}_n)).$$

$$((l-1)|\mathcal{P}| + 1) \text{ to } l|\mathcal{P}| \text{ in } \mathcal{A}^{\mathcal{P}} \rightarrow \mathcal{A}_l^{\mathcal{P}}$$

Estimation model

$$\begin{bmatrix} \Re\{\mathbf{i}\} \\ \Im\{\mathbf{i}\} \end{bmatrix} = \begin{bmatrix} \mathcal{A}^T \mathbf{C} & -\mathcal{A}^T \mathbf{D} & -\mathbf{E} \\ \mathcal{A}^T \mathbf{D} & \mathcal{A}^T \mathbf{C} & \mathbf{F} \end{bmatrix} \begin{bmatrix} \mathbf{X}^G \\ \mathbf{X}^B \\ \mathbf{X}^T \end{bmatrix},$$

$$\begin{bmatrix} \Re\{\mathbf{i}_{\mathcal{L}}\} \\ \Im\{\mathbf{i}_{\mathcal{L}}\} \end{bmatrix} = \begin{bmatrix} \mathbf{C} & -\mathbf{D} \\ \mathbf{D} & \mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{X}^G \\ \mathbf{X}^B \end{bmatrix}$$

$$\mathbf{C} = \text{diag}_{l \in \mathcal{L}}(\mathcal{A}_l^{\mathcal{P}} \Re\{\Lambda(\mathbf{v})\})$$

$$\mathbf{D} = \text{diag}_{l \in \mathcal{L}}(\mathcal{A}_l^{\mathcal{P}} \Im\{\Lambda(\mathbf{v})\})$$

$$\mathbf{E} = \text{diag}_{n \in \mathcal{N}}(\Im\{f_Y(\mathbf{v}_n)\})$$

$$\mathbf{F} = \text{diag}_{n \in \mathcal{N}}(\Re\{f_Y(\mathbf{v}_n)\})$$

$$\mathbf{X}^G = \text{col}_{l \in \mathcal{L}}(\Re\{\Xi(\mathbf{Y}_l)\})$$

$$\mathbf{X}^B = \text{col}_{l \in \mathcal{L}}(\Im\{\Xi(\mathbf{Y}_l)\})$$

$$\mathbf{X}^T = \text{col}_{t \in \mathcal{T}}(\Im\{\Xi(\mathbf{Y}_t)\}).$$

R.K. Gupta, F. Sossan, J.-Y., Le Boudec, and M. Paolone, "Compound admittance matrix estimation of three-phase untransposed power distribution grids using synchrophasor measurements", IEEE Transactions on Instrumentation and Measurement, 70, pp.1-13. 2021.



Estimation techniques

$$\mathcal{J}(k) = \mathcal{H}(k)\mathcal{X}$$

$$\text{where } \mathcal{H}(k) = \begin{bmatrix} \mathcal{A}^T \mathbf{C}(k) & -\mathcal{A}^T \mathbf{D}(k) & -\mathbf{E}(k) \\ \mathcal{A}^T \mathbf{D}(k) & \mathcal{A}^T \mathbf{C}(k) & \mathbf{F}(k) \end{bmatrix}, \mathcal{J}(k) = \begin{bmatrix} \Re\{\mathbf{i}(k)\} \\ \Im\{\mathbf{i}(k)\} \end{bmatrix}, \text{ and } \mathcal{X} = \begin{bmatrix} \mathbf{X}^G \\ \mathbf{X}^B \\ \mathbf{X}^T \end{bmatrix}$$

K measurements



$$\mathcal{J} = \mathcal{H}\mathcal{X}$$

- The choice of estimation scheme depends on the **characterization of noise** on $\mathcal{J}$ and $\mathcal{H}$.

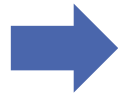
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Estimation Model

$$\mathcal{J} = \mathcal{H}\mathcal{X}$$

- Assuming that noise on $\mathcal{H}$ is negligible and noise on $\mathcal{J}$ is Gaussian and I.I.D.



Least
squares or
WLS



$$\mathcal{X}_{LS}^* = (\mathcal{H}^T \mathcal{H})^{-1} \mathcal{H}^T \mathcal{J}$$

- Assuming that noise on $\mathcal{H}$ and $\mathcal{J}$ are non-negligible and are Gaussian I.I.D



Total Least
squares



$$\mathcal{X}_{TLS}^* = (\mathcal{H}^T \mathcal{H} - \underline{\sigma}^2 I)^{-1} \mathcal{H}^T \mathcal{J}$$

$\underline{\sigma}$ is smallest eigen value of $[\mathcal{H} | \mathcal{J}]$

- The estimation performance depends on the **characterization of noise** on $\mathcal{J}$ and $\mathcal{H}$

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Noise Model

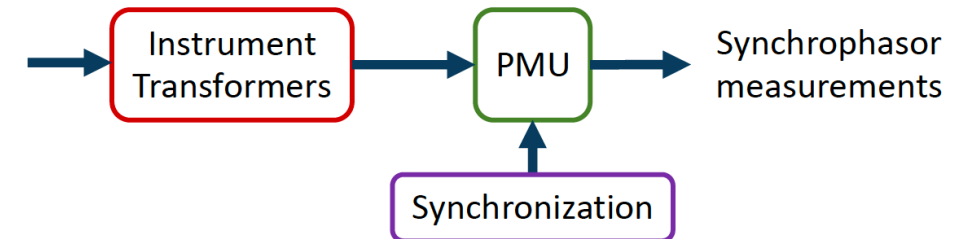
Noise model (polar coordinates)

- Sources of uncertainties
PMUs

- Magnitude max error $\alpha = 0.1\%$ and Phase max error $\beta = 10^{-4}$ rad.

Additive noise from Instrument Transformer (IT)

- White noise distributions for magnitudes $\Delta_p \sim \mathcal{N}\left(0, \frac{\alpha}{3}\right)$ and phases $\Delta_\phi \sim \mathcal{N}\left(0, \frac{\beta}{3}\right)$



Transformer Class	Max. magnitude error (α) [%]	Max. phase error (β) [rad]
0.1	0.1	1.5×10^{-3}
0.2	0.2	3×10^{-3}
0.5	0.5	9×10^{-3}
1	1	18×10^{-3}

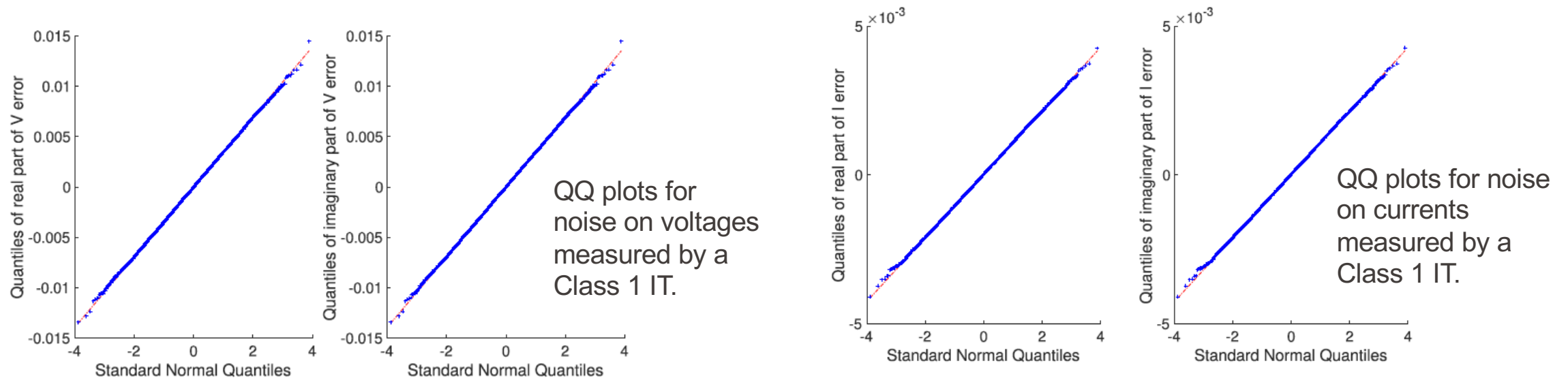
Instrument Transformers: „Additional requirements for electronic voltage transformers,” Standard IEC, pp. 61 869–8, 2011.
Instrument Transformers: „Additional requirements for electronic current transformers,” Standard IEC, pp. 61 869–7, 2014.



Noisy Model (Rectangular)

Noise model (rectangular coordinates)

- Note that the noise model is normal in polar coordinates, whereas we use rectangular coordinates in our estimation.
- **The noise transformation from polar to rectangular coordinates is nonlinear, and hence, it does not preserve the normality of the distribution.**
- However, for the typical uncertainties of ITs and PMUs, we may numerically observe that the distribution of the noise transformed in the rectangular coordinates is very close to be normal.

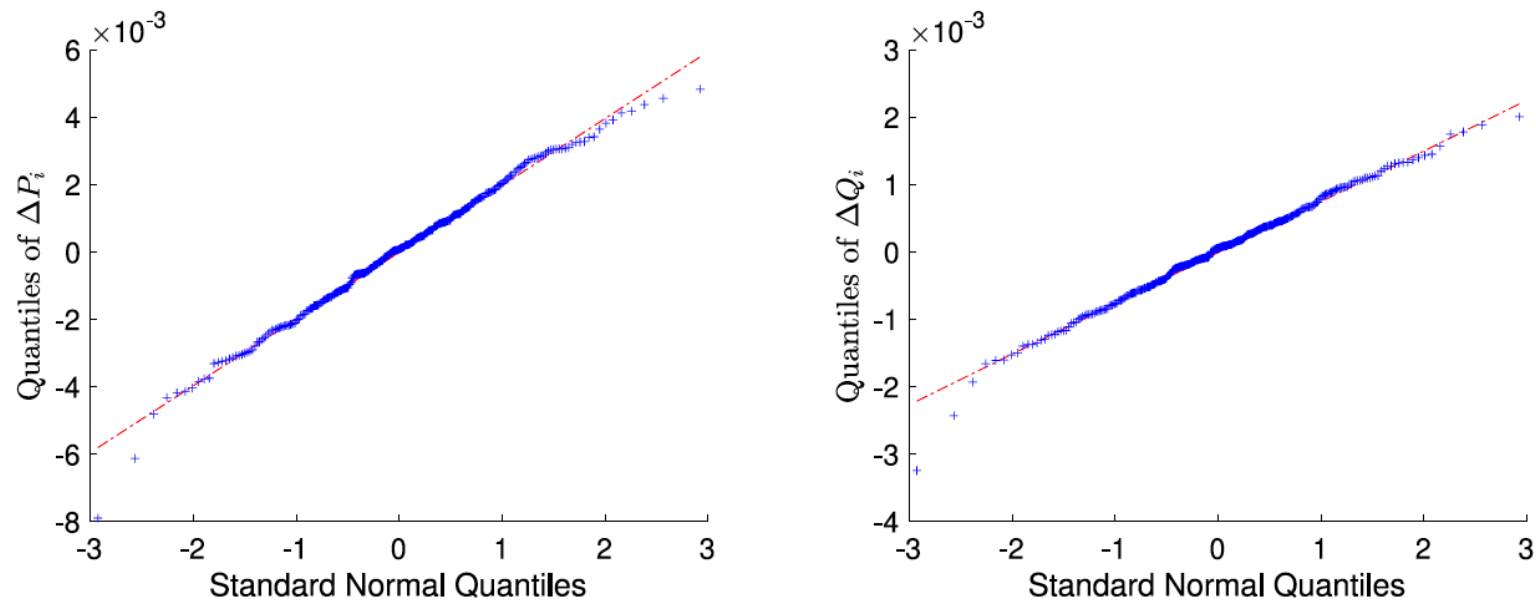


M. Paolone et al., "Static and recursive PMU-based State Estimation Processes for transmission and distribution grids", in the book "Advanced Techniques for Power System Modelling, Control and Stability Analysis", IET 2015.



Noisy Power Measurements

- For **power measurements** as they are non-linearly related to the voltages and currents and if the power measurements are derived from the voltage and current measurements, the resulting noise on the power measurements may not be Gaussian.



Gupta, R., Sossan, F., & Paolone, M. (2022). Model-less robust voltage control in active distribution networks using sensitivity coefficients estimated from measurements. *Electric Power Systems Research*, 212, 108547.



Noise variances for Rectangular coordinates

Noise model (rectangular coordinates)

To find the mean and covariances of polar-to-rectangular coordinates, let first denote the noisy measurement of the phasor $w = \rho e^{j\phi}$ as $\tilde{w} = \tilde{\rho} e^{j\tilde{\phi}}$. If Δ_{re} and Δ_{im} , respectively, denote the noise in the real and imaginary parts, then it can be shown using moment generating functions that:

$$\begin{bmatrix} \mathbf{X}^G \\ \mathbf{X}^B \end{bmatrix} = (\mathcal{H}^T \mathcal{W} \mathcal{H})^{-1} \mathcal{H}^T \mathcal{W} \mathcal{J}$$

$$W(t) = \begin{bmatrix} \boxed{} & & \\ & \boxed{} & \\ & & \ddots \\ & & & \boxed{} \end{bmatrix}$$

$$= \begin{bmatrix} E[\Delta_{Ire}^2] & E[\Delta_{Ire}, \Delta_{Iim}] \\ E[\Delta_{Ire}, \Delta_{Iim}] & E[\Delta_{Iim}^2] \end{bmatrix}$$

$$\begin{aligned} \Delta_{re} &= \tilde{\rho} \cos(\tilde{\phi}) - \rho \cos(\phi) \\ E[\Delta_{re}] &= (e^{-\frac{1}{2}\sigma_{\phi}^2} - 1) \cos(\phi) \rho \\ E[\Delta_{re}^2] &= \frac{1}{2} \left(1 + \frac{\alpha^2}{9} \right) \rho^2 (1 + e^{-2\sigma_{\phi}^2} \cos(2\phi)) \\ &\quad + \rho^2 \cos^2(\phi) (1 - 2e^{-\frac{1}{2}\sigma_{\phi}^2}) \\ \Delta_{im} &= \tilde{\rho} \sin(\tilde{\phi}) - \rho \sin(\phi) \\ E[\Delta_{im}] &= (e^{-\frac{1}{2}\sigma_{\phi}^2} - 1) \sin(\phi) \rho \\ E[\Delta_{im}^2] &= \frac{1}{2} \left(1 + \frac{\alpha^2}{9} \right) \rho^2 (1 + e^{-2\sigma_{\phi}^2} \cos(2\phi)) \\ &\quad + \rho^2 \sin^2(\phi) (1 - 2e^{-\frac{1}{2}\sigma_{\phi}^2}) \\ E[\Delta_{re} \Delta_{im}] &= \frac{1}{2} \sin(2\phi) \left[\left(1 + \frac{\alpha^2}{9} \right) \rho^2 e^{-2\sigma_{\phi}^2} \right. \\ &\quad \left. - 2\rho^2 e^{-\frac{1}{2}\sigma_{\phi}^2} + \rho^2 \right] \end{aligned}$$

Book chapter M. Paolone et al., "Static and recursive PMU-based State Estimation Processes for transmission and distribution grids", in the book "Advanced Techniques for Power System Modelling, Control and Stability Analysis", IET 2015.

D. Lerro and Y. Bar-Shalom, "Tracking with debiased consistent converted measurements versus EKF," IEEE Trans. Aerosp. Electron. Syst., vol. 29, no. 3, pp. 1015–1022, Jul. 1993.

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Estimation techniques

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K measurements



$$\mathcal{J} = \mathcal{H}\mathcal{X}$$

- The choice of estimation scheme depends on the **characterization of noise** on $\mathcal{J}$ and $\mathcal{H}$.

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Estimation Model

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- Assuming that noise on $\mathcal{H}$ is negligible and noise on $\mathcal{J}$ is Gaussian and I.I.D.

Least
squares or
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$$\mathcal{X}_{TLS}^* = (\mathcal{H}^T \mathcal{H} - \underline{\sigma}^2 I)^{-1} \mathcal{H}^T \mathcal{J}$$

$\underline{\sigma}$ is smallest eigen value of $[\mathcal{H}|\mathcal{J}]$

- The estimation performance depends on the **characterization of noise** on $\mathcal{J}$ and $\mathcal{H}$.

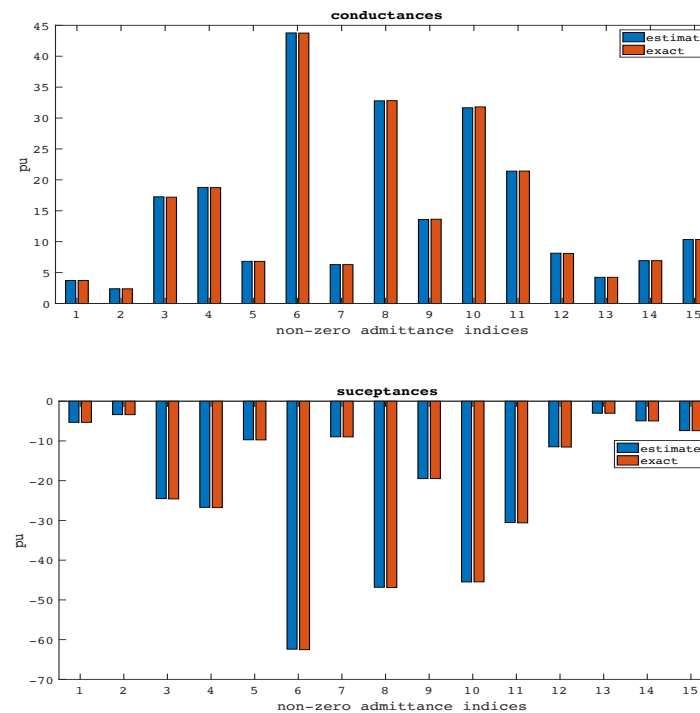
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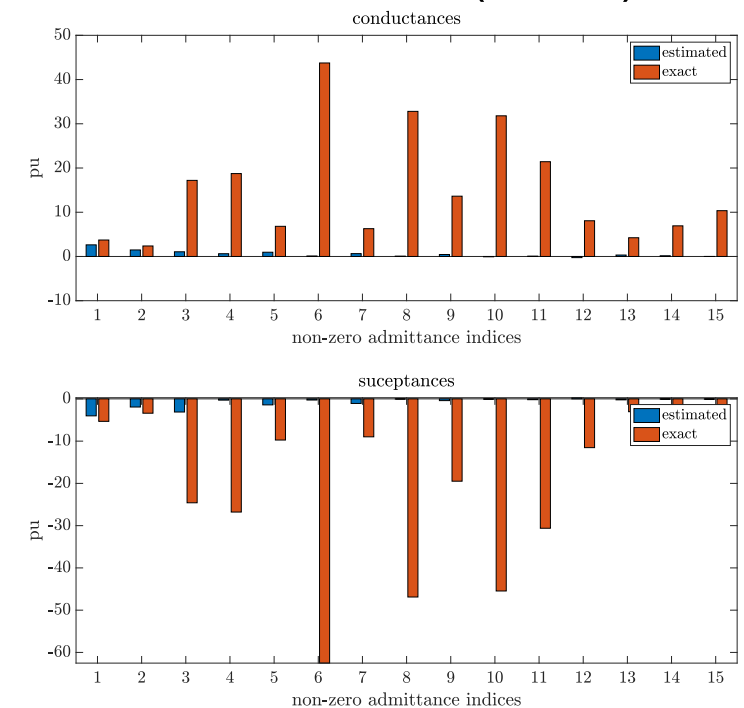
Estimation Performance with “Noise”

- With slight increase in the measurement noise, there is a large increase in the estimation error.

Without noise



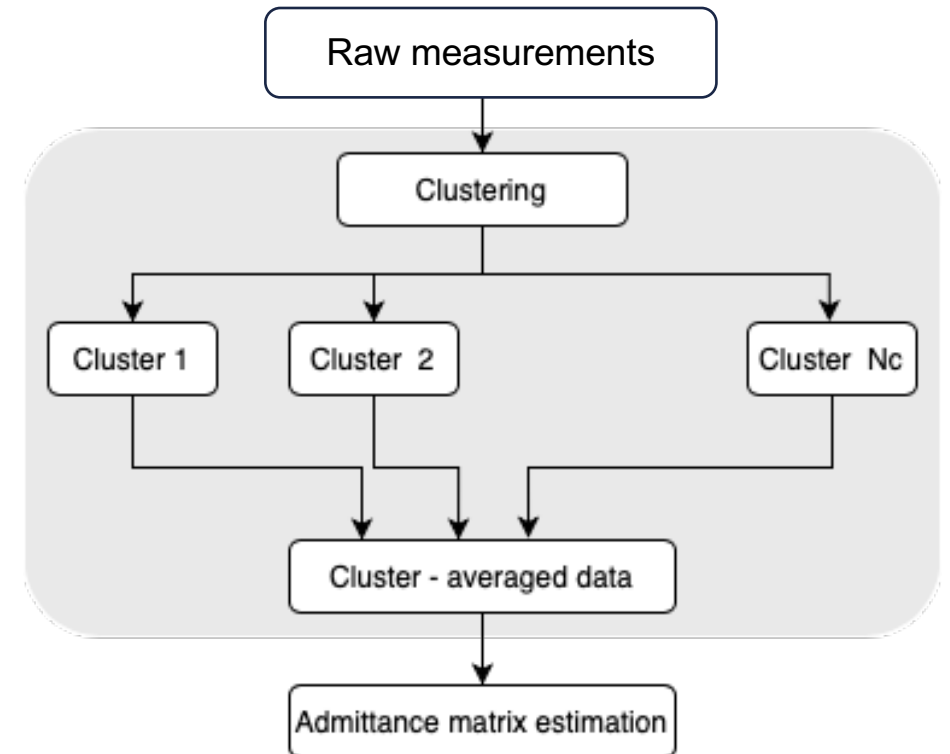
With noise (IT 0.5)



Proposed pre-processing: clustering averaging

- Measurements are clustered into different groups using the `k-means` algorithm.
- Averaged measurements from each cluster then used for estimation.

```
1: procedure CLUSTERAVG
2:   choose number of clusters =  $N_c$ 
3:    $[\{\mathbf{v}, \mathbf{i}\}_1, \dots, \{\mathbf{v}, \mathbf{i}\}_{N_c}] \leftarrow \text{kmeans}(\{\mathbf{v}, \mathbf{i}\}, \text{features}, N_c)$ 
4:   for  $l = 1 : N_c$  do
5:      $\{\tilde{\mathbf{v}}, \tilde{\mathbf{i}}\}_l = \text{mean}(\{\mathbf{v}, \mathbf{i}\}_l)$ 
6:   end for
7: end procedure
```



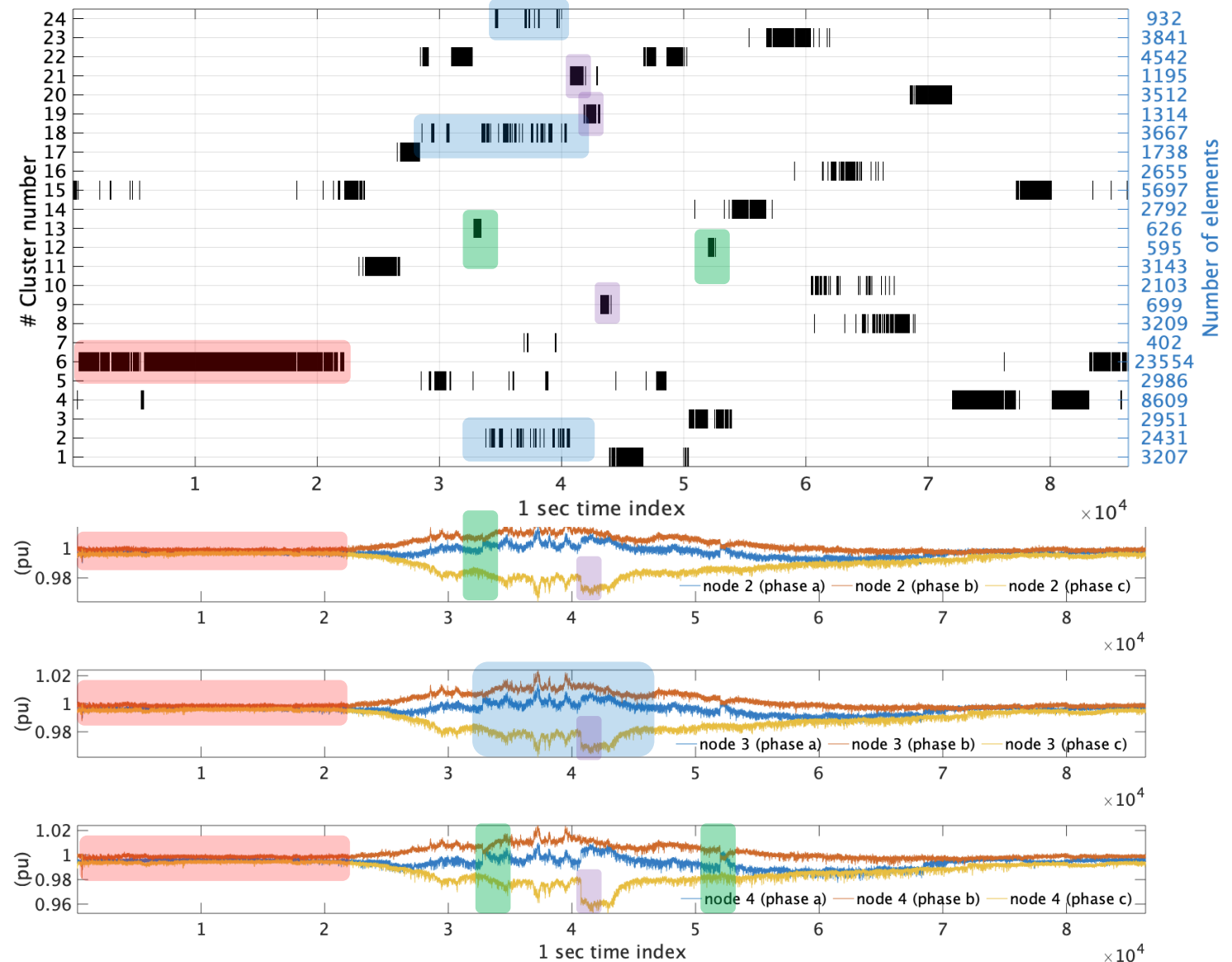
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Proposed pre-processing: clustering averaging

Example

- **Clusters 6:** balanced conditions cluster together
- **Cluster 9, 19, 21:** sudden dip in nodal voltages
- **Clusters 12, 13:** voltage dip
- **Clusters 2, 18, 24:** sharp change in PV generation



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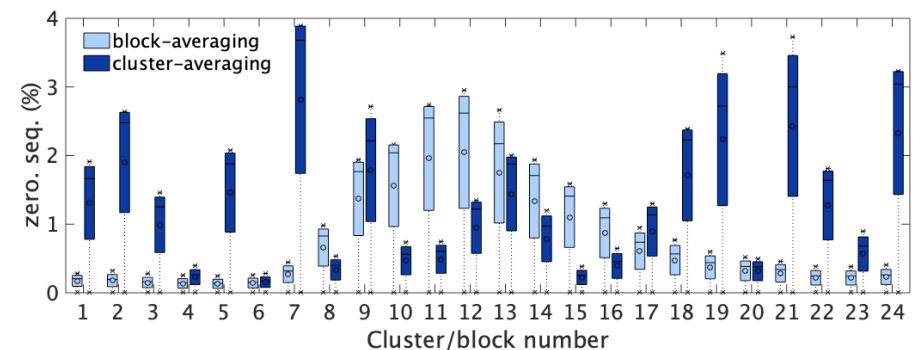
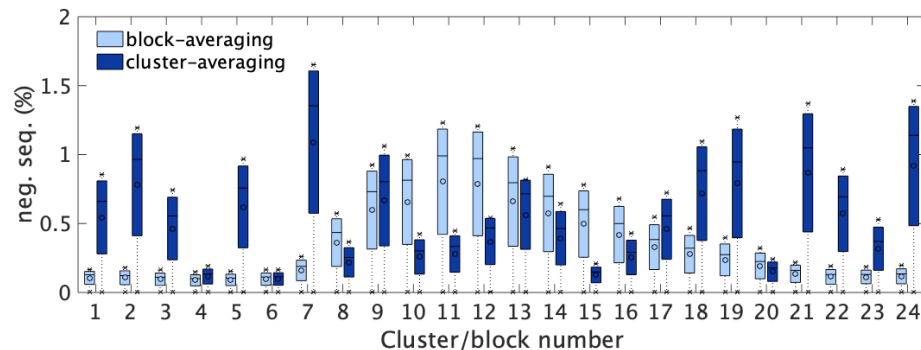
Proposed pre-processing: clustering averaging

- **Cluster avg.** provides higher values of positive and negative seq. components compared to **block-avg.** helps in estimating the off-diagonal elements of the lines and shunt admittances.

Example

$$\text{metrics} \quad \frac{|v_{\text{neg}}|}{|v_{\text{pos}}|} \times 100\% \quad \frac{|v_{\text{zero}}|}{|v_{\text{pos}}|} \times 100\%$$

Method	Neg. seq. (%)		Zero seq. (%)	
	mean	max	mean	max
Block-averaging	0.32	0.80	0.69	2.04
Cluster-averaging	0.49	1.71	1.15	4.05



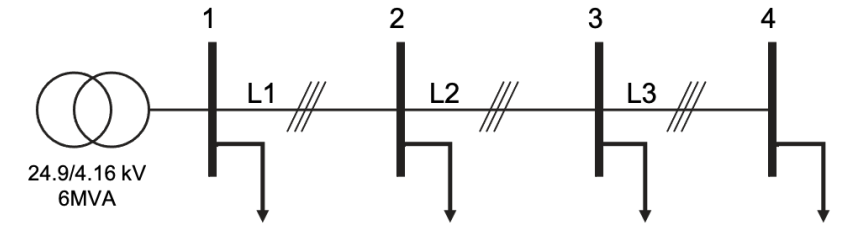
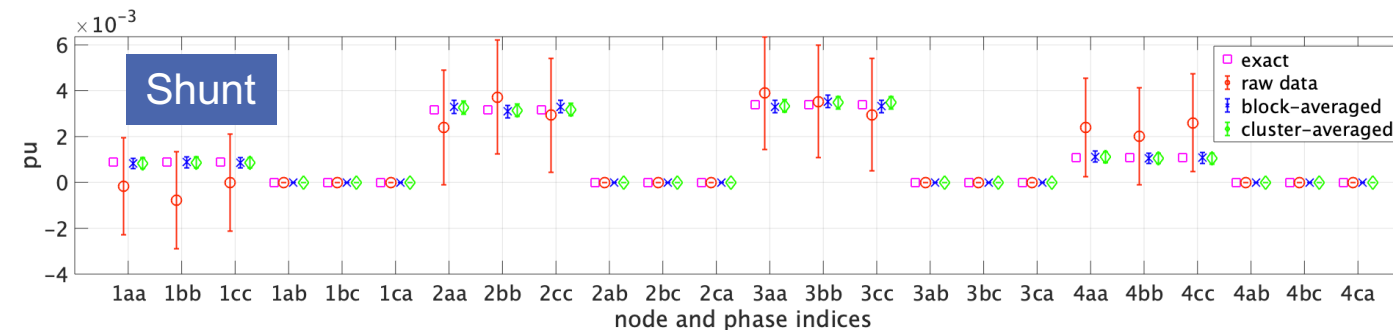
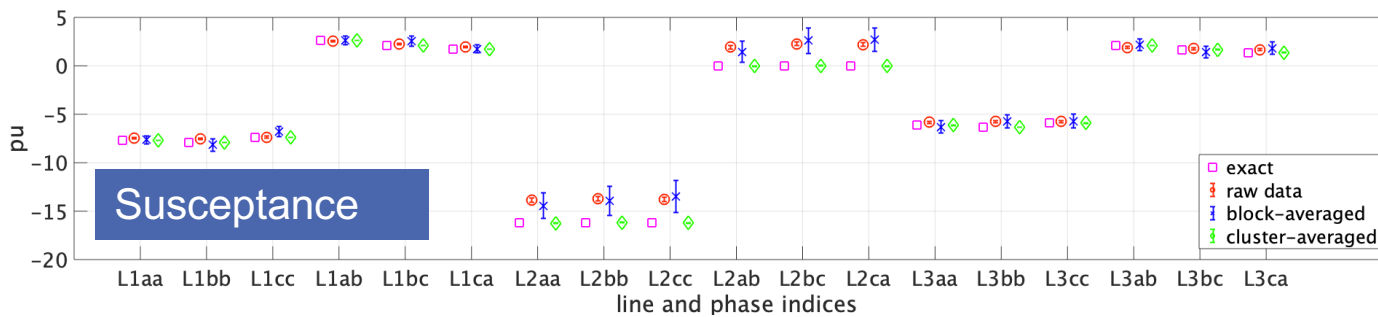
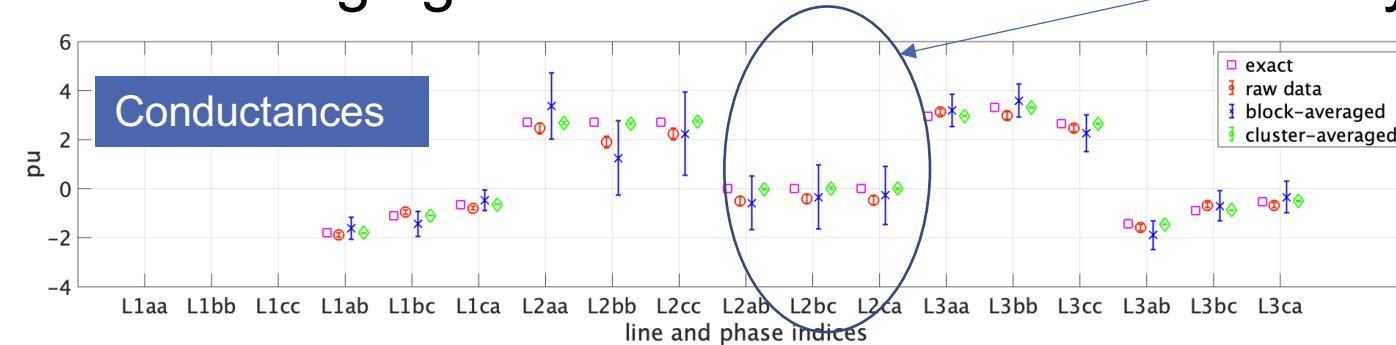
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Estimation results

Exact value 0

- Cluster-averaging shows better estimation accuracy (mean values and variances).



$$\text{NMSE} = \mathcal{E}(\mathbf{X}^*, \hat{\mathbf{X}}) = \mathcal{E}_{\mathbf{X}} = \frac{\|\hat{\mathbf{X}} - \mathbf{X}^*\|_2}{\|\hat{\mathbf{X}}\|_2}$$

$$\sigma_{\mathbf{X}} = \sqrt{(\sigma_r^2 \text{diag}((\mathbf{H}^T \mathbf{H})^{-1}))}$$

Test-networks	Raw-data		Cluster-averaging	
	LS	TLS	LS	TLS
CIGRE MV (BT)	0.897	6.137	0.016	0.012
CIGRE LV (BT)	0.620	5.877	0.002	0.002
CIGRE LV (UU)	0.371	4.77	0.033	0.037
CIGRE microgrid (BT)	0.884	6.812	0.044	0.019
IEEE13 (UU)	0.585	2.961	0.060	0.056
IEEE123 (BT)	0.946	15.551	0.060	0.060



Performance sensitivity

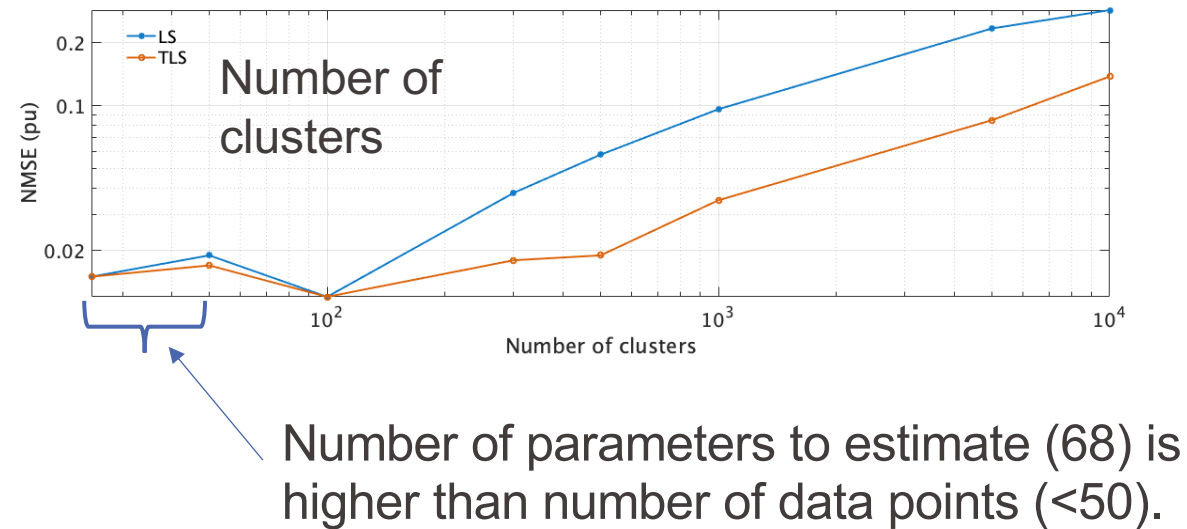
Different
noise levels

IT class	Raw-data		Cluster-averaging	
	LS	TLS	LS	TLS
0.1	0.064	0.012	0.002	0.002
0.2	0.181	0.038	0.003	0.003
0.5	0.404	0.148	0.010	0.009
1	0.548	9.171	0.018	0.014

Different
clustering
features

Clustering features	LS	TLS
Voltage magnitude	0.036	0.024
Current magnitude	0.021	0.021
Voltage and current magnitude	0.010	0.009
Sequence voltage magnitude	0.066	0.432
Sequence current magnitude	0.054	0.056
Sequence voltage and current magnitude	0.019	0.018

Normalized MSE



Estimating State-dependent Parameters

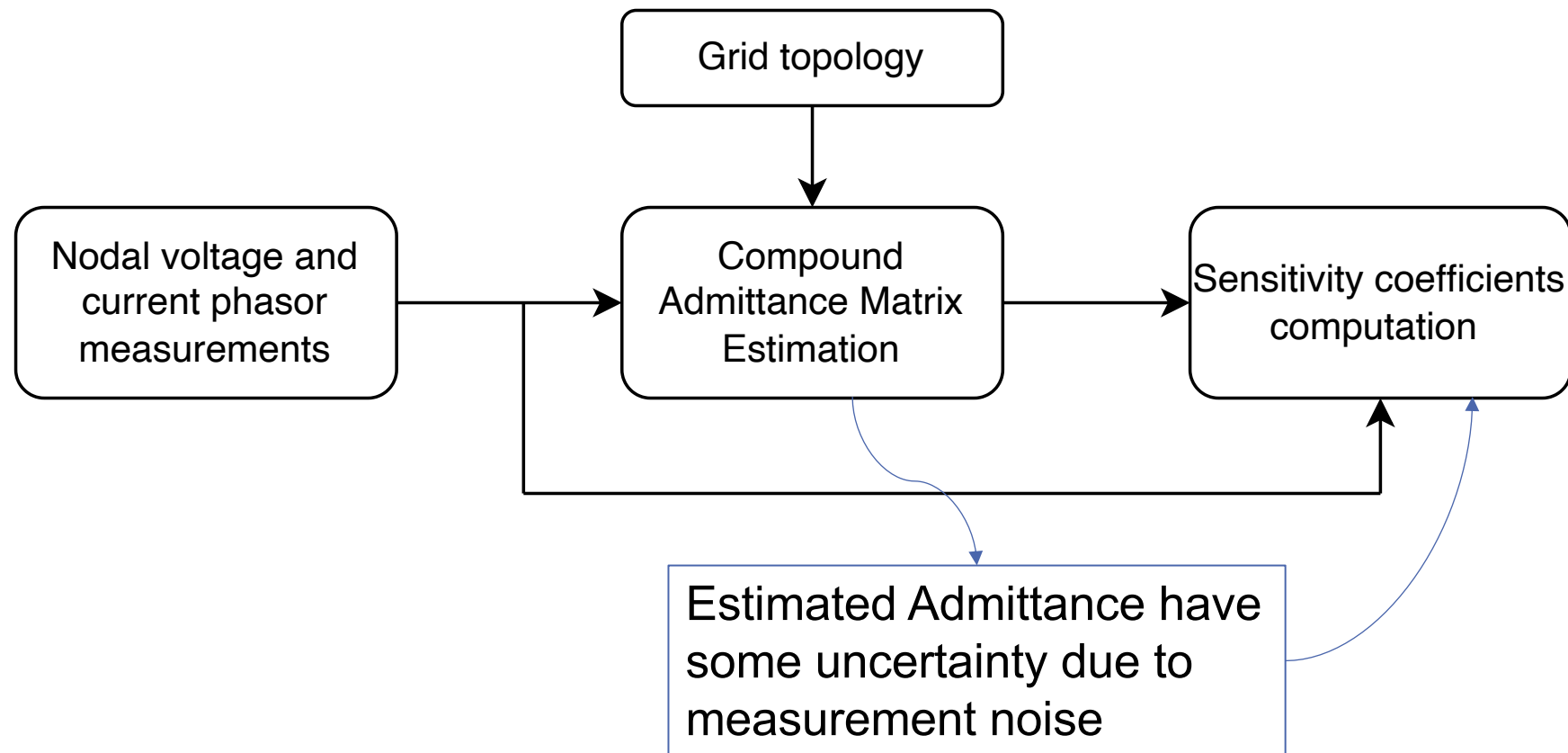
- Estimating Voltage Sensitivity coefficient
- Measurement-based voltage control

$$K_{ij}^p = \frac{\partial |v_i|}{\partial p_j}; K_{ij}^q = \frac{\partial |v_i|}{\partial q_j}$$

K_{ij}^p K_{ij}^q are the sensitivity coefficient for i -th nodal voltage magnitudes w.r.t. power injections at node j .



Scheme1: Using Estimated Y to Compute Sensitivities



- A mechanism to propagate the uncertainty from uncertain Y to sensitivity coefficient.



Estimating Sensitivity Coefficients from Y

$$\begin{aligned}
 \underline{s}_i &= p_i - jq_i = \underline{v}_i \sum_{j \in \mathcal{N}} Y_{ij} v_j \\
 \Downarrow \\
 \mathbb{1}_{\{i=l\}} &= \frac{\partial \underline{v}_i}{\partial p_l} \sum_{j \in \mathcal{N}} Y_{ij} v_j + \underline{v}_i \sum_{j \in \mathcal{N}} Y_{ij} \frac{\partial v_j}{\partial p_l}, \\
 -j \mathbb{1}_{\{i=l\}} &= \frac{\partial \underline{v}_i}{\partial q_l} \sum_{j \in \mathcal{N}} Y_{ij} v_j + \underline{v}_i \sum_{j \in \mathcal{N}} Y_{ij} \frac{\partial v_j}{\partial q_l}. \\
 \Downarrow \\
 \mathbf{z} &= \mathbf{H} \mathbf{x} \\
 \Downarrow \\
 \mathbf{x} &= \mathbf{H}^{-1} \mathbf{z}. \\
 \Downarrow \\
 \frac{\partial |v_l|}{\partial p_l} &= \frac{1}{|v_l|} \Re \left(\underline{v}_l \frac{\partial v_l}{\partial p_l} \right)
 \end{aligned}$$

$$\begin{aligned}
 H_{11} &= v_1^{re} Y_{21}^{re} - v_1^{im} Y_{21}^{im} - v_3^{im} Y_{32}^{im} - v_2^{re} + 2v_2^{re} Y_{22}^{re} + v_3^{re} Y_{32}^{re}, \\
 H_{12} &= v_2^{im} Y_{32}^{im} + v_2^{re} Y_{32}^{re}, \\
 H_{13} &= v_1^{im} Y_{21}^{re} + Y_{21}^{im} v_1^{re} + 2v_2^{im} Y_{22}^{re} + v_3^{im} Y_{32}^{re} + Y_{32}^{im} v_3^{re}, \\
 H_{14} &= v_2^{im} Y_{32}^{re} - Y_{32}^{im} v_2^{re}, \\
 H_{21} &= v_3^{im} Y_{32}^{im} + v_3^{re} Y_{32}^{re}, \\
 H_{22} &= v_1^{re} Y_{31}^{re} - v_1^{im} Y_{31}^{im} - v_2^{im} Y_{32}^{im} - v_3^{re} + v_2^{re} Y_{32}^{re} + 2v_3^{re} Y_{33}^{re}, \\
 H_{23} &= v_3^{im} Y_{32}^{re} - Y_{32}^{im} v_3^{re}, \\
 H_{24} &= v_1^{im} Y_{31}^{re} + Y_{31}^{im} v_1^{re} + v_2^{im} Y_{32}^{re} + Y_{32}^{im} v_2^{re} + 2v_3^{im} Y_{33}^{re}, \\
 H_{31} &= v_1^{im} Y_{21}^{re} + Y_{21}^{im} v_1^{re} + 2Y_{22}^{im} v_2^{re} + v_3^{im} Y_{32}^{re} + Y_{32}^{im} v_3^{re}, \\
 H_{32} &= Y_{32}^{im} v_2^{re} - v_2^{im} Y_{32}^{re}, \\
 H_{33} &= v_1^{im} Y_{21}^{im} + 2v_2^{im} Y_{22}^{im} + v_3^{im} Y_{32}^{im} - v_1^{re} Y_{21}^{re} - v_3^{re} Y_{32}^{re}, \\
 H_{34} &= v_2^{im} Y_{32}^{im} + v_2^{re} Y_{32}^{re}, \\
 H_{41} &= Y_{32}^{im} v_3^{re} - v_3^{im} Y_{32}^{re}, \\
 H_{42} &= v_1^{im} Y_{31}^{re} + Y_{31}^{im} v_1^{re} + v_2^{im} Y_{32}^{re} + Y_{32}^{im} v_2^{re} + 2Y_{33}^{im} v_3^{re}, \\
 H_{43} &= v_3^{im} Y_{32}^{im} + v_3^{re} Y_{32}^{re}, \\
 H_{44} &= v_1^{im} Y_{31}^{im} + v_2^{im} Y_{32}^{im} + 2v_3^{im} Y_{33}^{im} - v_1^{re} Y_{31}^{re} - v_2^{re} Y_{32}^{re}.
 \end{aligned}$$

Both $\mathbf{Y}$ and $\mathbf{v}$ elements are uncertain

$$\mathbf{z} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Three-node system example

Uncertainty Propagation from inverse of H

- Matrix $\mathbf{H}$ is uncertain, to compute uncertainty on $\mathbf{x} = \mathbf{H}^{-1}\mathbf{z}$.

$$[\sigma_{\mathbf{x}}]_i^2 \approx [\mathbf{H}^{-1}]_{ij}^2 [\sigma_{\mathbf{z}}]_j^2 + [\sigma_{\mathbf{H}^{-1}}]_{ij}^2 [\mathbf{z}]_j^2. \quad \rightarrow \quad [\sigma_{\mathbf{x}}]_i^2 = [\sigma_{\mathbf{H}^{-1}}]_{ij}^2 [\mathbf{z}]_j^2.$$

$$\begin{aligned} \text{cov}(\mathbf{H}_{mn}^{-1}, \mathbf{H}_{mn}^{-1}) &= \sum_i \sum_j [\mathbf{H}^{-1}]_{mi}^2 [\sigma_{\mathbf{H}}]_{ij}^2 [\mathbf{H}^{-1}]_{jn}^2 \\ \text{cov}(\mathbf{H}_{mn}^{-1}, \mathbf{H}_{ab}^{-1}) &= \sum_i \sum_j [\mathbf{H}^{-1}]_{mi} [\mathbf{H}^{-1}]_{ai} [\sigma_{\mathbf{H}}]_{ij}^2 [\mathbf{H}^{-1}]_{jn} [\mathbf{H}^{-1}]_{jb} \end{aligned}$$

Propagation rules for additive and multiplicative expression

$$\sigma(\Theta + \Phi)^2 \approx \sigma_{\Theta}^2 + \sigma_{\Phi}^2 + 2\sigma_{\Theta\Phi},$$

$$\sigma(\Theta \times \Phi)^2 \approx \Theta^2 \Phi^2 (\sigma_{\Theta}^2 / \Theta^2 + \sigma_{\Phi}^2 / \Phi^2 + 2\sigma_{\Theta\Phi} / \Theta\Phi).$$

Example

$$\mathbf{H} = \begin{bmatrix} \epsilon & \zeta \\ \eta & \xi \end{bmatrix}$$

$$\sigma_{\mathbf{H}}^2 = \begin{bmatrix} \sigma_{\epsilon}^2 & \sigma_{\zeta}^2 \\ \sigma_{\eta}^2 & \sigma_{\xi}^2 \end{bmatrix}$$

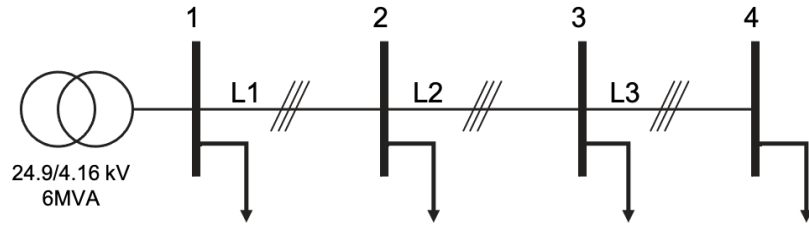
$$\begin{aligned} \text{cov}([\mathbf{H}^{-1}]_{11}, [\mathbf{H}^{-1}]_{ij}) &= \begin{bmatrix} \epsilon^4 \sigma_{\epsilon}^2 + \epsilon^2 \zeta^2 \sigma_{\eta}^2 + \epsilon^2 \eta^2 \sigma_{\delta}^2 + \zeta^2 \eta^2 \sigma_{\xi}^2 & \epsilon^3 \zeta \sigma_{\epsilon}^2 + \xi \eta \epsilon^2 \sigma_{\delta}^2 + \epsilon \zeta^3 \sigma_{\eta}^2 + \xi \eta \zeta^2 \sigma_{\xi}^2 \\ \epsilon^3 \eta \sigma_{\epsilon}^2 + \zeta \xi \epsilon^2 \sigma_{\eta}^2 + \epsilon \eta^3 \sigma_{\delta}^2 + \zeta \xi \eta^2 \sigma_{\xi}^2 & \epsilon^2 \zeta \eta \sigma_{\epsilon}^2 + \epsilon \zeta^2 \xi \sigma_{\eta}^2 + \epsilon \xi \eta^2 \sigma_{\delta}^2 + \zeta \xi^2 \eta \sigma_{\xi}^2 \end{bmatrix} \\ \text{cov}([\mathbf{H}^{-1}]_{12}, [\mathbf{H}^{-1}]_{ij}) &= \begin{bmatrix} \epsilon^3 \zeta \sigma_{\epsilon}^2 + \xi \eta \epsilon^2 \sigma_{\delta}^2 + \epsilon \zeta^3 \sigma_{\eta}^2 + \xi \eta \zeta^2 \sigma_{\xi}^2 & \epsilon^2 \zeta^2 \sigma_{\epsilon}^2 + \epsilon^2 \xi^2 \sigma_{\delta}^2 + \zeta^4 \sigma_{\eta}^2 + \zeta^2 \xi^2 \sigma_{\xi}^2 \\ \epsilon^2 \zeta \eta \sigma_{\epsilon}^2 + \epsilon \zeta^2 \xi \sigma_{\eta}^2 + \epsilon \xi \eta^2 \sigma_{\delta}^2 + \zeta \xi^2 \eta \sigma_{\xi}^2 & \zeta^3 \xi \sigma_{\eta}^2 + \epsilon \eta \zeta^2 \sigma_{\epsilon}^2 + \zeta \xi^3 \sigma_{\xi}^2 + \epsilon \eta \xi^2 \sigma_{\delta}^2 \end{bmatrix} \\ \text{cov}([\mathbf{H}^{-1}]_{21}, [\mathbf{H}^{-1}]_{ij}) &= \begin{bmatrix} \epsilon^3 \eta \sigma_{\epsilon}^2 + \zeta \xi \epsilon^2 \sigma_{\eta}^2 + \epsilon \eta^3 \sigma_{\delta}^2 + \zeta \xi \eta^2 \sigma_{\xi}^2 & \epsilon^2 \zeta \eta \sigma_{\epsilon}^2 + \epsilon \zeta^2 \xi \sigma_{\eta}^2 + \epsilon \xi \eta^2 \sigma_{\delta}^2 + \zeta \xi^2 \eta \sigma_{\xi}^2 \\ \epsilon^2 \xi^2 \sigma_{\eta}^2 + \epsilon^2 \eta^2 \sigma_{\epsilon}^2 + \xi^2 \eta^2 \sigma_{\delta}^2 + \eta^4 \sigma_{\xi}^2 & \xi^3 \eta \sigma_{\xi}^2 + \epsilon \zeta \xi^2 \sigma_{\eta}^2 + \xi \eta^3 \sigma_{\delta}^2 + \epsilon \zeta \eta^2 \sigma_{\epsilon}^2 \end{bmatrix} \\ \text{cov}([\mathbf{H}^{-1}]_{22}, [\mathbf{H}^{-1}]_{ij}) &= \begin{bmatrix} \epsilon^2 \zeta \eta \sigma_{\epsilon}^2 + \epsilon \zeta^2 \xi \sigma_{\eta}^2 + \epsilon \xi \eta^2 \sigma_{\delta}^2 + \zeta \xi^2 \eta \sigma_{\xi}^2 & \zeta^3 \xi \sigma_{\eta}^2 + \epsilon \eta \zeta^2 \sigma_{\epsilon}^2 + \zeta \xi^3 \sigma_{\xi}^2 + \epsilon \eta \xi^2 \sigma_{\delta}^2 \\ \xi^3 \eta \sigma_{\xi}^2 + \epsilon \zeta \xi^2 \sigma_{\eta}^2 + \xi \eta^3 \sigma_{\delta}^2 + \epsilon \zeta \eta^2 \sigma_{\epsilon}^2 & \zeta^2 \xi^2 \sigma_{\eta}^2 + \zeta^2 \eta^2 \sigma_{\epsilon}^2 + \xi^4 \sigma_{\xi}^2 + \xi^2 \eta^2 \sigma_{\delta}^2 \end{bmatrix} \end{aligned}$$

Gupta, R. K. (2023). Quantifying the Uncertainty of Sensitivity Coefficients Computed From Uncertain Compound Admittance Matrix and Noisy Grid Measurements. *IEEE Transactions on Instrumentation and Measurement*, 73, 1-4.

M. Lefebvre, R. Keeler, R. Sobie, and J. White, "Propagation of errors for matrix inversion," *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, vol. 451, no. 2, pp. 520–528, 2000.



Estimating Sensitivities and Uncertainties from Y



$$\begin{aligned}
 H_{11} &= v_1^{re} Y_{21}^{re} - v_1^{im} Y_{21}^{im} - v_3^{im} Y_{32}^{im} - v_2^{re} + 2v_2^{re} Y_{22}^{re} + v_3^{re} Y_{32}^{re}, \\
 H_{12} &= v_2^{im} Y_{32}^{im} + v_2^{re} Y_{32}^{re}, \\
 H_{13} &= v_1^{im} Y_{21}^{re} + Y_{21}^{im} v_1^{re} + 2v_2^{im} Y_{22}^{re} + v_3^{im} Y_{32}^{re} + Y_{32}^{im} v_3^{re}, \\
 H_{14} &= v_2^{im} Y_{32}^{re} - Y_{32}^{im} v_2^{re}, \\
 H_{21} &= v_3^{im} Y_{32}^{im} + v_3^{re} Y_{32}^{re}, \\
 H_{22} &= v_1^{re} Y_{31}^{re} - v_1^{im} Y_{31}^{im} - v_2^{im} Y_{32}^{im} - v_3^{re} + v_2^{re} Y_{32}^{re} + 2v_3^{re} Y_{33}^{re}, \\
 H_{23} &= v_3^{im} Y_{32}^{re} - Y_{32}^{im} v_3^{re}, \\
 H_{24} &= v_1^{im} Y_{31}^{re} + Y_{31}^{im} v_1^{re} + v_2^{im} Y_{32}^{re} + Y_{32}^{im} v_2^{re} + 2v_3^{im} Y_{33}^{re}, \\
 H_{31} &= v_1^{im} Y_{21}^{re} + Y_{21}^{im} v_1^{re} + 2Y_{22}^{im} v_2^{re} + v_3^{im} Y_{32}^{re} + Y_{32}^{im} v_3^{re}, \\
 H_{32} &= Y_{32}^{im} v_2^{re} - v_2^{im} Y_{32}^{re}, \\
 H_{33} &= v_1^{im} Y_{21}^{im} + 2v_2^{im} Y_{22}^{im} + v_3^{im} Y_{32}^{im} - v_1^{re} Y_{21}^{re} - v_3^{re} Y_{32}^{re}, \\
 H_{34} &= v_2^{im} Y_{32}^{im} + v_2^{re} Y_{32}^{re}, \\
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 H_{42} &= v_1^{im} Y_{31}^{re} + Y_{31}^{im} v_1^{re} + v_2^{im} Y_{32}^{re} + Y_{32}^{im} v_2^{re} + 2Y_{33}^{im} v_3^{re}, \\
 H_{43} &= v_3^{im} Y_{32}^{im} + v_3^{re} Y_{32}^{re}, \\
 H_{44} &= v_1^{im} Y_{31}^{im} + v_2^{im} Y_{32}^{im} + 2v_3^{im} Y_{33}^{im} - v_1^{re} Y_{31}^{re} - v_2^{re} Y_{32}^{re}.
 \end{aligned}$$

$$\text{cov}(\mathbf{H}_{mn}^{-1}, \mathbf{H}_{mn}^{-1}) = \sum_i \sum_j [\mathbf{H}^{-1}]_{mi}^2 [\sigma_{\mathbf{H}}]_{ij}^2 [\mathbf{H}^{-1}]_{jn}^2$$

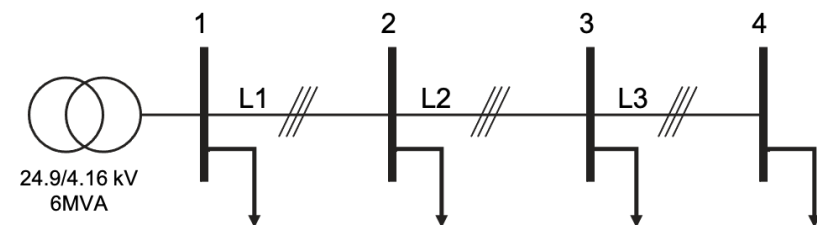
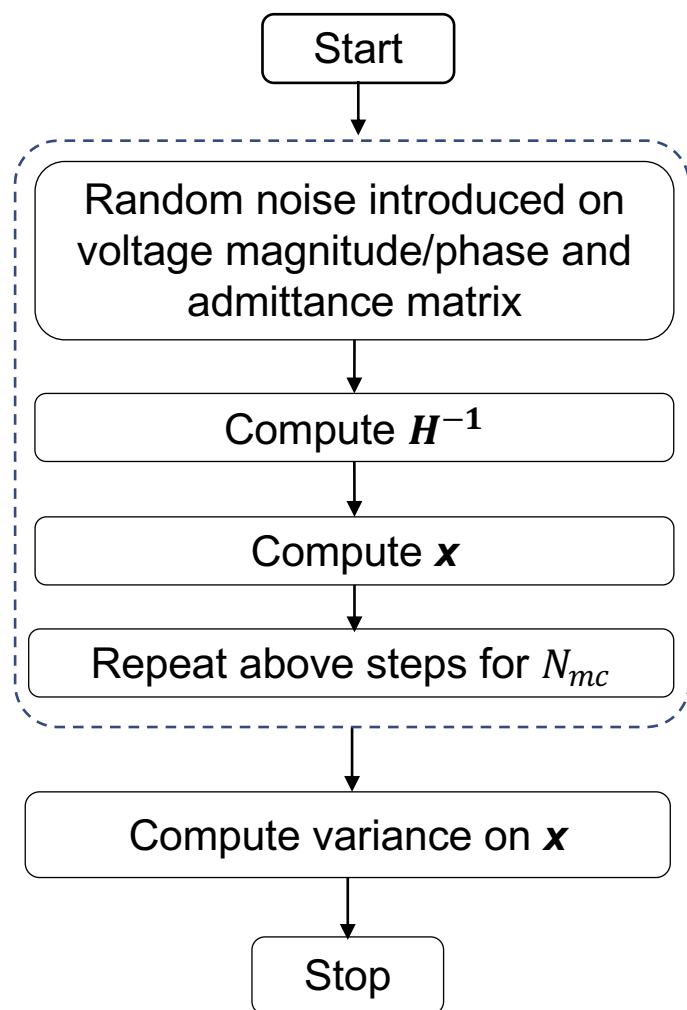
$$\text{cov}(\mathbf{H}_{mn}^{-1}, \mathbf{H}_{ab}^{-1}) = \sum_i \sum_j [\mathbf{H}^{-1}]_{mi} [\mathbf{H}^{-1}]_{ai} [\sigma_{\mathbf{H}}]_{ij}^2 [\mathbf{H}^{-1}]_{jn} [\mathbf{H}^{-1}]_{jb}$$

	$\Re(\frac{\partial v_3}{\partial p_2})$	$\Re(\frac{\partial v_3}{\partial p_4})$	$\Re(\frac{\partial v_4}{\partial p_4})$	$\Im(\frac{\partial v_3}{\partial p_2})$	$\Im(\frac{\partial v_3}{\partial p_3})$	$\Im(\frac{\partial v_4}{\partial p_4})$
Nominal value	0.0071	0.0152	0.0239	0.0077	0.0176	0.0288
Variance (Analytical)	0.0006	0.0012	0.0018	0.0006	0.0011	0.0016

Gupta, R. K. (2023). Quantifying the Uncertainty of Sensitivity Coefficients Computed From Uncertain Compound Admittance Matrix and Noisy Grid Measurements. *IEEE Transactions on Instrumentation and Measurement*, 73, 1-4.



Monte Carlo Validation



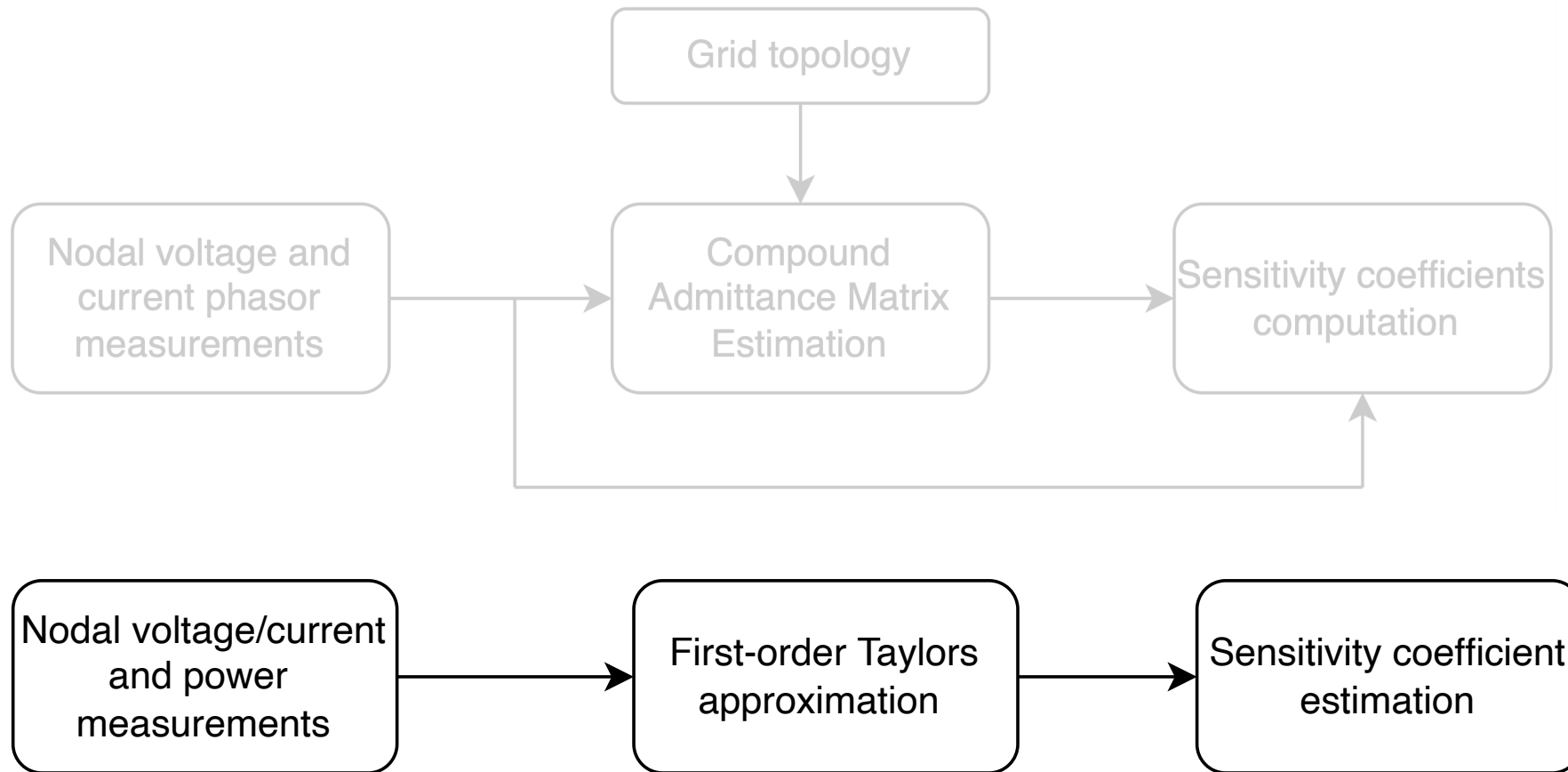
		$\Re(\frac{\partial v_3}{\partial p_2})$	$\Re(\frac{\partial v_3}{\partial p_4})$	$\Re(\frac{\partial v_4}{\partial p_4})$	$\Im(\frac{\partial v_3}{\partial p_2})$	$\Im(\frac{\partial v_3}{\partial p_3})$	$\Im(\frac{\partial v_4}{\partial p_4})$
	Nominal value	0.0071	0.0152	0.0239	0.0077	0.0176	0.0288
	Variance (Analytical)	0.0006	0.0012	0.0018	0.0006	0.0011	0.0016
	N_{mc}						
Variance (Monte-Carlo)	10	0.0009	0.0016	0.0021	0.0006	0.0012	0.0018
	100	0.0007	0.0013	0.0020	0.0007	0.0013	0.0019
	1000	0.0007	0.0014	0.0020	0.0007	0.0013	0.0019
	10000	0.0007	0.0014	0.0020	0.0007	0.0013	0.0019

Analytical propagation is coherent with MC.

Gupta, R. K. (2023). Quantifying the Uncertainty of Sensitivity Coefficients Computed From Uncertain Compound Admittance Matrix and Noisy Grid Measurements. *IEEE Transactions on Instrumentation and Measurement*, 73, 1-4.



Scheme2: Using Taylor's Approximation



Sensitivity Estimation (Scheme2)

▪ Hypothesis:

- Sensitivity coefficients remain **unchanged till next control actuation**,
- System is in steady-state and have **small dynamics**: First-order Taylor's approximation is sufficiently accurate, and
- The **magnitude error noise** of the volt-meter measurements is **Gaussian** and Independent and identically distributed (**I.I.D**)

$$\underbrace{|\Delta v|_{i,t_k}}_{\gamma_{t_k}} \approx \underbrace{[\Delta p_{t_k} \Delta q_{t_k}]}_{h_{t_k}} \underbrace{\begin{bmatrix} K_{i,t_k}^p \\ K_{i,t_k}^q \end{bmatrix}}_X$$

Deviations of the voltage from previous timestep

Deviations of the active/reactive powers

Sensitivity coefficients

$$\Gamma = \mathbf{H}\mathbf{X} + \mathcal{W} \quad \mathcal{W} \in \mathcal{N}(\mathbf{0}, \Sigma),$$

$$\mathbf{\Gamma} \in \mathbb{R}^N = [\gamma_{t_1} \gamma_{t_2} \dots \gamma_{t_N}]^T$$

$$\mathbf{H} \in \mathbb{R}^{N \times 2N_b} = [h_{t_1} h_{t_2} \dots h_{t_N}]^T$$

Objective is to estimate $\mathbf{X}$ using measurements contained in $\mathbf{H}$ matrix.

Gupta, R., Sossan, F., & Paolone, M. (2022). Model-less robust voltage control in active distribution networks using sensitivity coefficients estimated from measurements. *Electric Power Systems Research*, 212, 108547.



Recursive Estimation

- Using Least Squares with quadratic regularization term (**Ridge regression**), λ is positive weight to penalize high values

$$\hat{\mathbf{X}} = \min_{\mathbf{X}} \|\mathbf{\Gamma} - \mathbf{H}\mathbf{X}\|_2 + \lambda \mathbf{X}^T \mathbf{X}$$



$$\hat{\mathbf{X}}_{t_0} = (\mathbf{H}^T \mathbf{H} + \lambda \mathbf{I})^{-1} \mathbf{H}^T \mathbf{\Gamma} = (\mathbf{R}_{t_0} + \lambda \mathbf{I}) \mathbf{H}^T \mathbf{\Gamma}$$

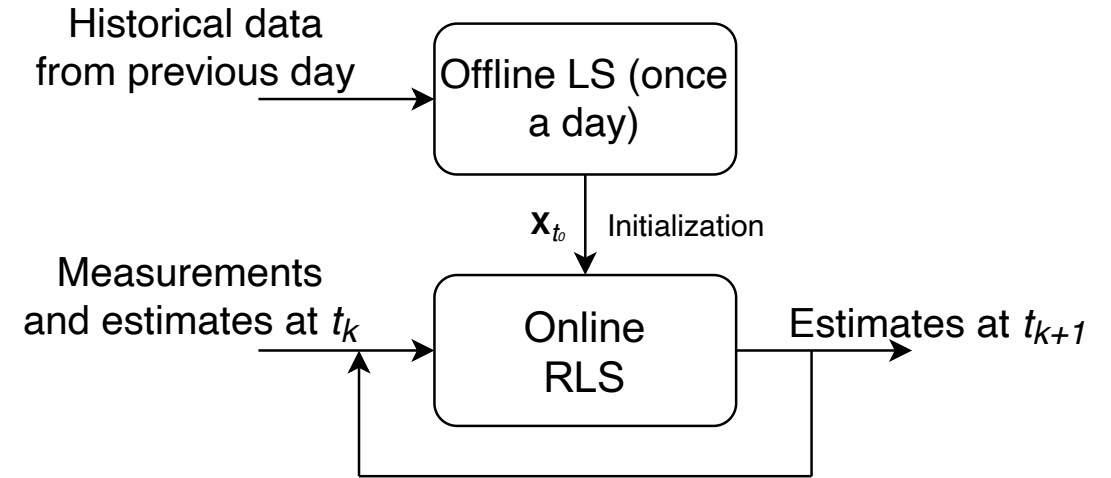
- LS suffers from multiple problems
 - Bad estimation in case of low excitation.
 - High bias with high measurement noise.
 - Performs poorly with multi-collinear measurements, when power injections at different nodes are similar.
- **With state changing, it might make sense to use most recent estimates applying recursive methods.**

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Recursive Least Squares

- **LS** for (**Offline**) rough estimates
- **Online estimation** of sensitivity coefficient is proposed using **Recursive Least Squares** where LS estimates are refined using most recent measurements. A forgetting factor μ is used to partially forget previous estimates.



$$\mathbf{R}_{t_k} = \mu \mathbf{R}_{t_{k-1}} + h_{t_k}^T h_{t_k}$$

Recursive estimation of the covariance matrix using recent measurements

$$e_{t_k} = \gamma_{t_k} - h_{t_k} \hat{\mathbf{X}}_{t_{k-1}}$$

$$\hat{\mathbf{X}}_{t_k} = \hat{\mathbf{X}}_{t_{k-1}} + \mathbb{G}_{t_k} e_{t_k}$$

$$\mathbb{G}_{t_k} = \frac{\mathbf{P}_{t_{k-1}}^{\text{cov}} h_{t_k}^T}{\mu + h_{t_k} \mathbf{P}_{t_{k-1}}^{\text{cov}} h_{t_k}^T}$$

$$\mathbf{P}_{t_k}^{\text{cov}} = (\mathbf{I} - \mathbb{G}_{t_k} h_{t_k}) \mathbf{P}_{t_{k-1}}^{\text{cov}} / \mu$$

RLS-F suffers from windup problem

Gupta, R., Sossan, F., & Paolone, M. (2022). Model-less robust voltage control in active distribution networks using sensitivity coefficients estimated from measurements. *Electric Power Systems Research*, 212, 108547.



Recursive Least Squares

Tackling windup problem

- **Constant-Trace scheme (RLS-CT):** setting an upper-bound on the trace sum of covariance matrix and adding offset.
- **Selective forgetting (RLS-SF):** using different forgetting factors for different eigenvalues of the covariance matrix. The factors are updated iteratively.
- **Directional forgetting (RLS-DF):** the covariance matrix $\mathbf{R}$ is split into orthogonal parts, first part is propagated whereas second with forgetting factor.

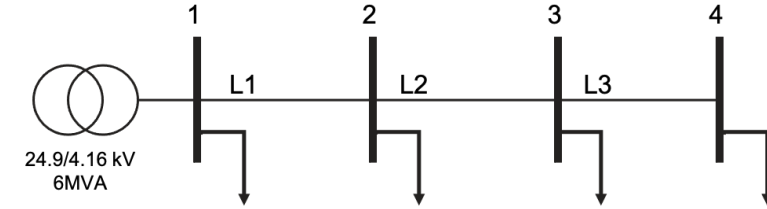
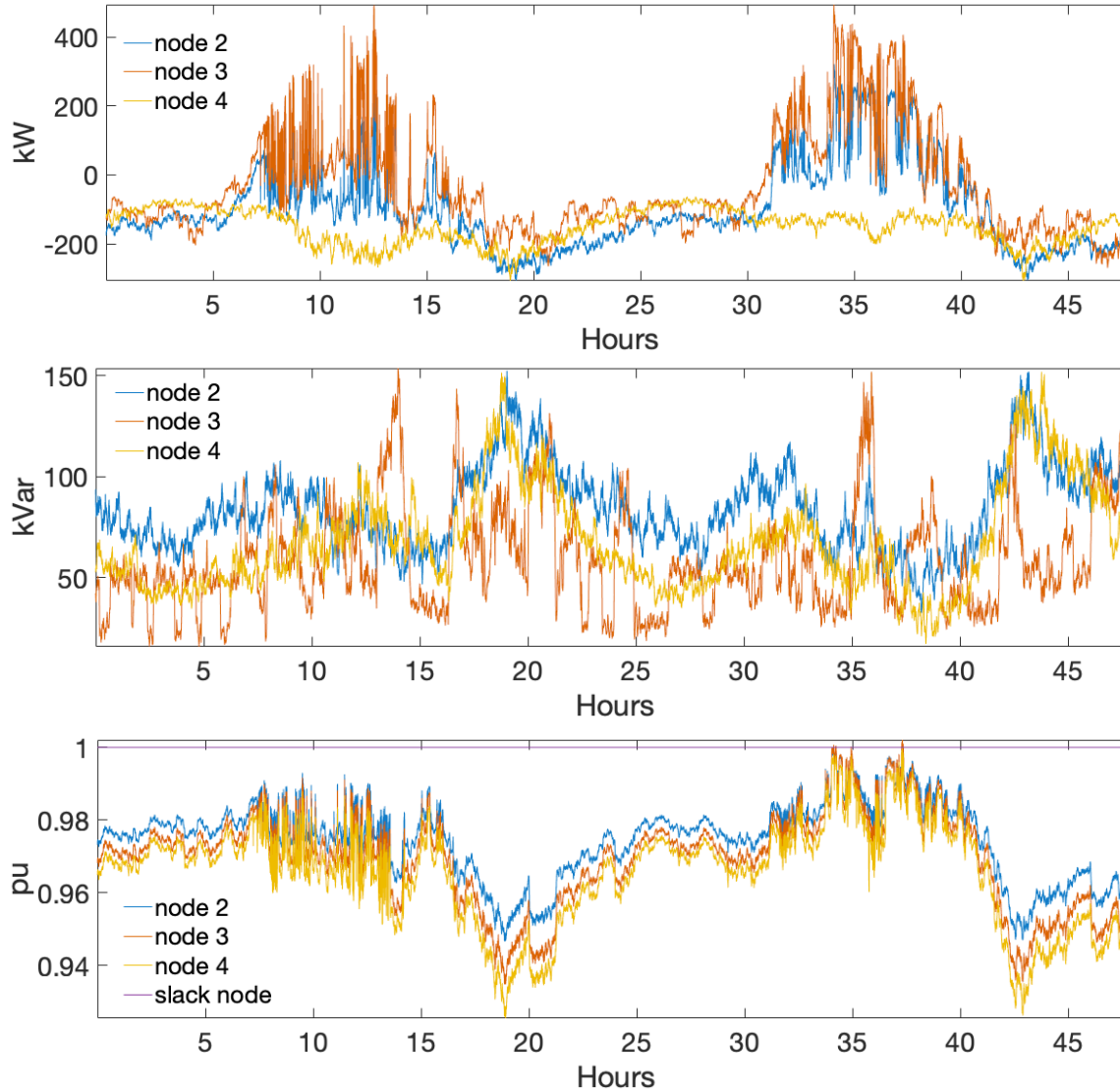
[RLC-CT] Fortescue, T.R., Kershenbaum, L.S. and Ydstie, B.E., 1981. Implementation of self-tuning regulators with variable forgetting factors. *Automatica*, 17(6), pp.831-835.

[RLS-SF] Parkum, J.E., Poulsen, N.K. and Holst, J., 1992. Recursive forgetting algorithms. *International Journal of Control*, 55(1), pp.109-128.

[RLS-DF] Cao, L. and Schwartz, H.M., 1999, June. A novel recursive algorithm for directional forgetting. In *Proceedings of the 1999 American Control Conference* (Cat. No. 99CH36251) (Vol. 2, pp. 1334-1338). IEEE.



Simulation Setup



$$\text{RMSE}(\hat{\mathbf{X}}) = \frac{\|\mathbf{X}^{\text{true}} - \hat{\mathbf{X}}\|_2}{\|\mathbf{X}^{\text{true}}\|_2}.$$

$$\text{PICP} = \frac{1}{T} \sum_{t_k=t_1}^{t_T} b_{t_k}$$

$$b_{t_k} = \begin{cases} 1 & \hat{K}_{ij,t_k}^p - \Delta K_{ij,t_k}^p \leq \hat{K}_{ij,t_k}^p \leq K_{ij,t_k}^p + \Delta K_{ij,t_k}^p \\ 0 & \text{otherwise.} \end{cases}$$

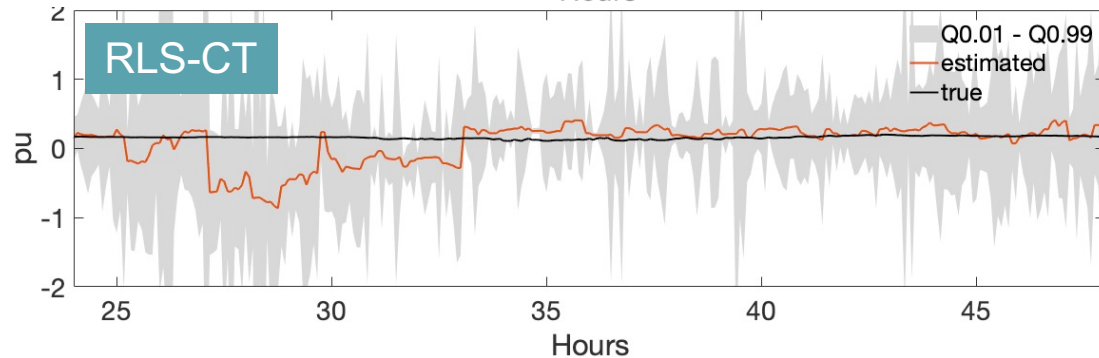
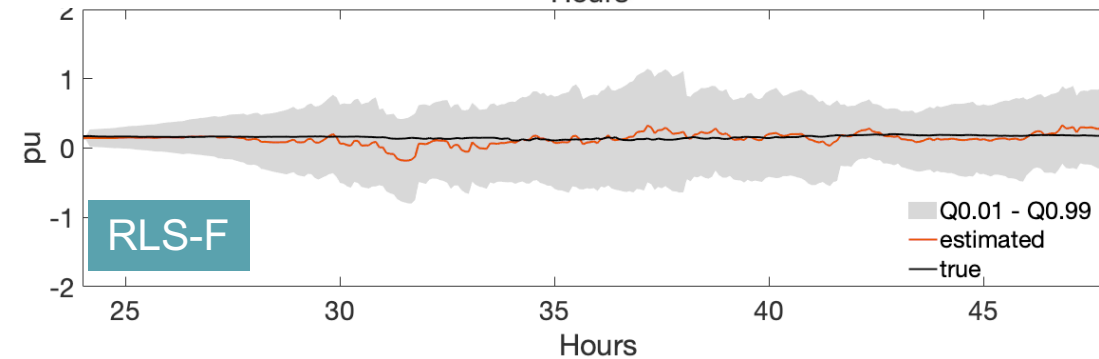
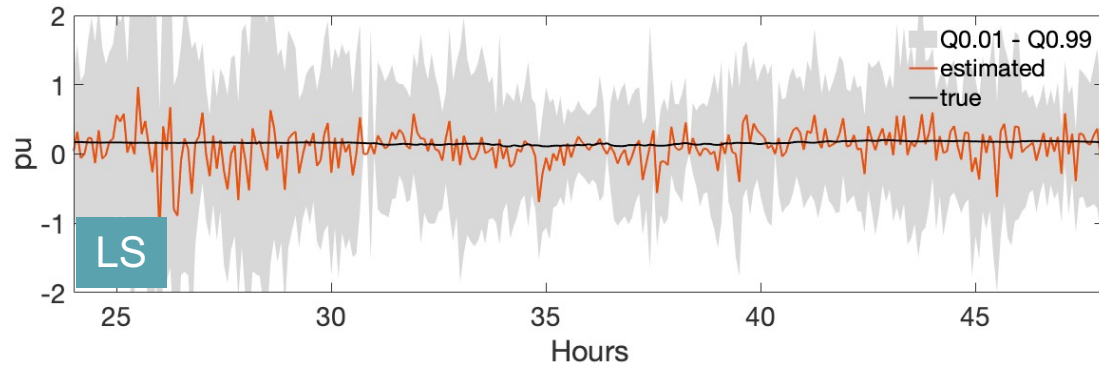
$$\text{PINAW} = \frac{1}{T(K_{ij,\max}^p)} \sum_{t_k=t_1}^{t_T} (2\Delta K_{ij,t_k}^p).$$

$$\text{CWC} = \text{PINAW}(1 + \eta(\text{PICP})e^{-(\nu(\text{PICP}-\text{CI}))})$$

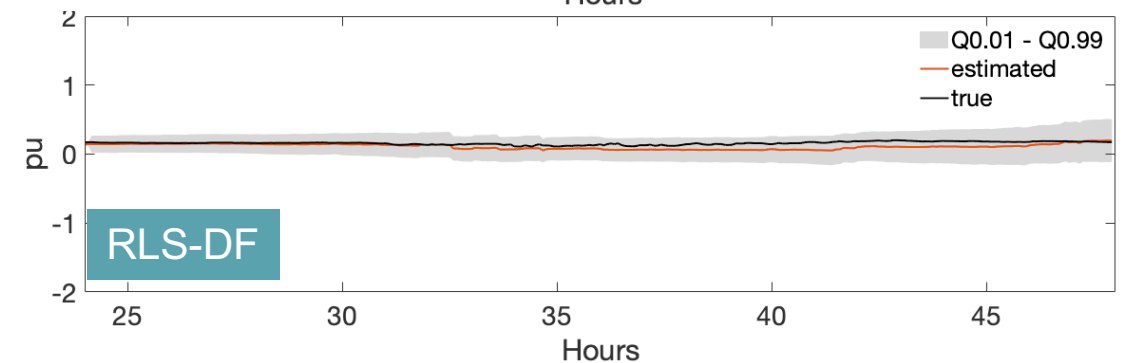
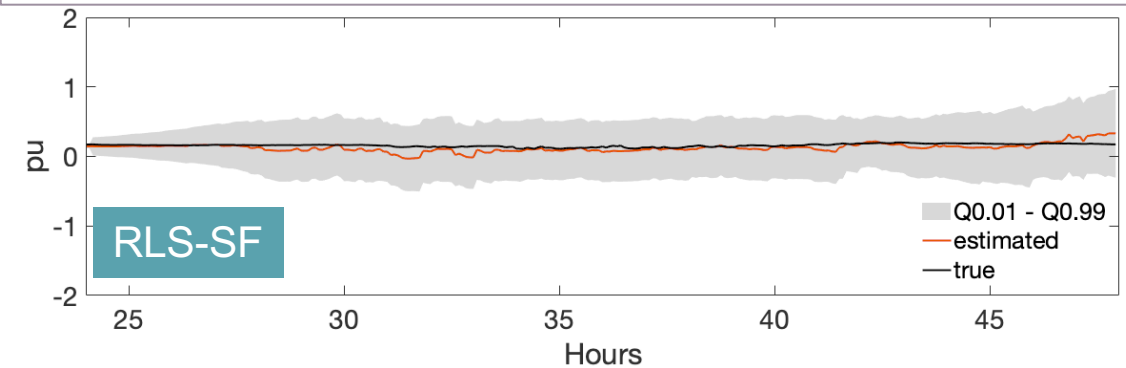
$$\eta = \begin{cases} 0, & \text{PICP} \leq \text{CI} \\ 1, & \text{otherwise} \end{cases}$$



Estimation Results ($K_{2,3}^p$)



- LS fails in reliably estimating the coefficients.
- RLS-F have very high variance (windup problem).
- RLS-CT does not have good coverage.
- RLS-SF and RLS-DF shows similar performances.



Gupta, R., Sossan, F., & Paolone, M. (2022). Model-less robust voltage control in active distribution networks using sensitivity coefficients estimated from measurements. *Electric Power Systems Research*, 212, 108547.



Performance with Different IT Class

- Measurement noise added from measuring instruments of different IT classes.

IT class	Voltage transformers [120]		Current transformers [119]	
	mag. error (σ^m) [%]	phase error σ^p [rad.]	mag. error (σ^m) [%]	phase error σ^p [rad.]
0.1	0.1	1.5e-3	0.1	1.5e-3
0.2	0.2	3e-3	0.2	3e-3
0.5	0.5	6e-3	0.5	9e-3
1	1	12e-3	1	18e-3

$K_{2,3}^p$

	IT 0.2		IT 0.5		IT 1.0	
Method	RMSE	PICP-PINAW-CWC	RMSE	PICP-PINAW-CWC	RMSE	PICP-PINAW-CWC
LS	0.97	0.93-4.21-7.81	2.58	0.93-9.73-18.04	8.78	0.69-17.07-28.19
RLS-F	0.31	0.99-5.08-5.08	1.03	0.99-7.49-7.49	1.98	1-13.71-13.71
RLS-CT	1.11	0.89-4.45-8.11	2.1	0.89-11.23-20.41	5.61	0.85-20.1-35.83
RLS-SF	0.2	0.99-2.39-2.39	0.69	0.99-5.74-5.74	1.55	1-10.78-10.78
RLS-DF	0.12	0.99-0.94-0.94	0.37	0.99-2.25-2.25	0.46	1-3.16-3.16

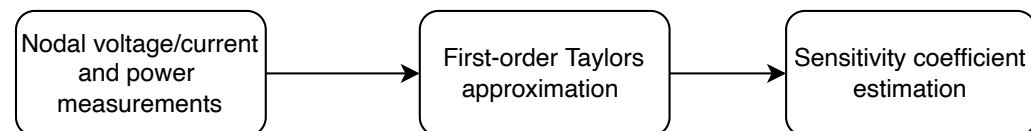
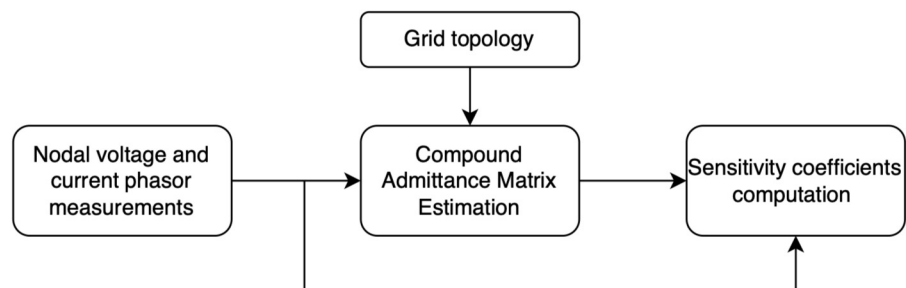
$K_{2,3}^q$

	IT 0.2		IT 0.5		IT 1.0	
Method	RMSE	PICP-PINAW-CWC	RMSE	PICP-PINAW-CWC	RMSE	PICP-PINAW-CWC
LS	0.78	0.99-4.97-4.97	1.05	1-8.49-8.49	1.93	0.99-10.76-10.76
RLS-F	0.29	0.99-3.23-3.23	0.46	0.99-4.69-4.69	0.98	0.99-8.32-8.32
RLS-CT	0.29	0.93-1.74-3.22	0.44	0.95-4.39-4.39	1.4	0.85-7.87-14.06
RLS-SF	0.15	0.98-1-1	0.24	0.99-2.4-2.4	0.61	0.99-4.52-4.52
RLS-DF	0.16	0.97-0.41-0.41	0.17	0.99-0.95-0.95	0.37	0.95-1.31-1.31

Gupta, R., Sossan, F., & Paolone, M. (2022). Model-less robust voltage control in active distribution networks using sensitivity coefficients estimated from measurements. *Electric Power Systems Research*, 212, 108547.



Scheme 1 vs. Scheme 2



IEEE 4 network

Noise level	RMSE	PICP-PINAW-CWC
IT 0.5	0.03	1-0.29-0.29
IT 1.0	0.06	1-0.59-0.59

Modified IEEE4 network – lines length increased by a factor of 6.

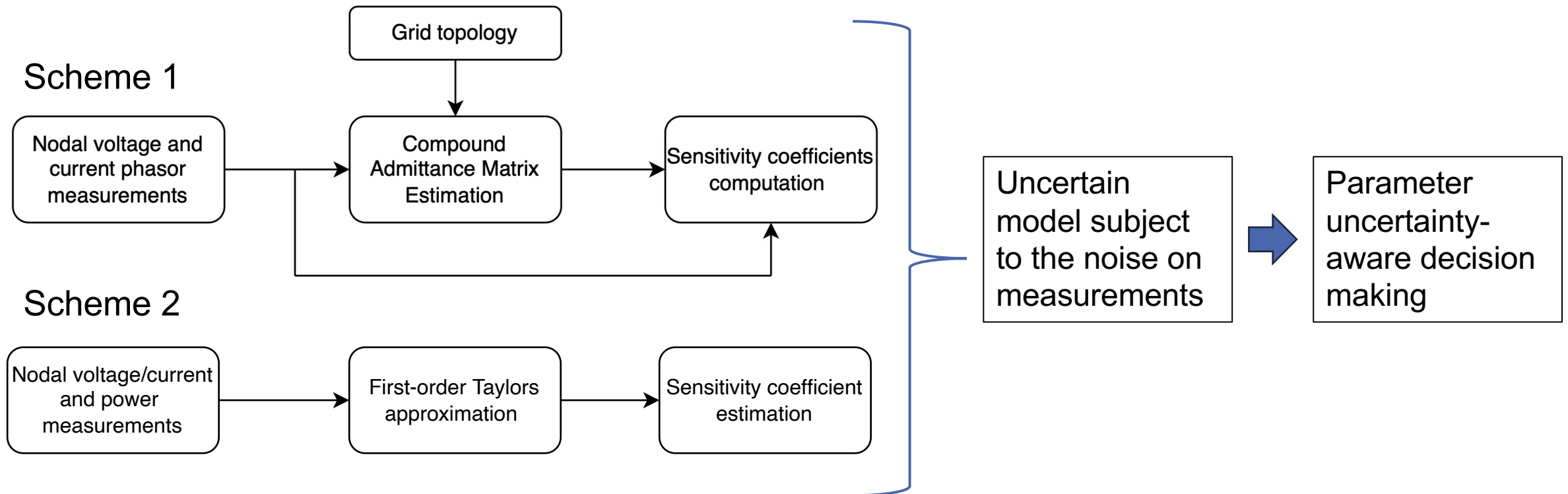
Noise level	RMSE	PICP-PINAW-CWC
IT 0.5	0.02	1-9.3-9.3
IT 1.0	0.05	1-19.25-19.25

Noise level	RMSE	PICP-PINAW-CWC
IT 0.5	0.29	1-1.85-1.85
IT 1.0	0.37	1-2.62-2.62

Noise level	RMSE	PICP-PINAW-CWC
IT 0.5	0.31	1-2.54-2.54
IT 1.0	0.98	1-3.46-3.46



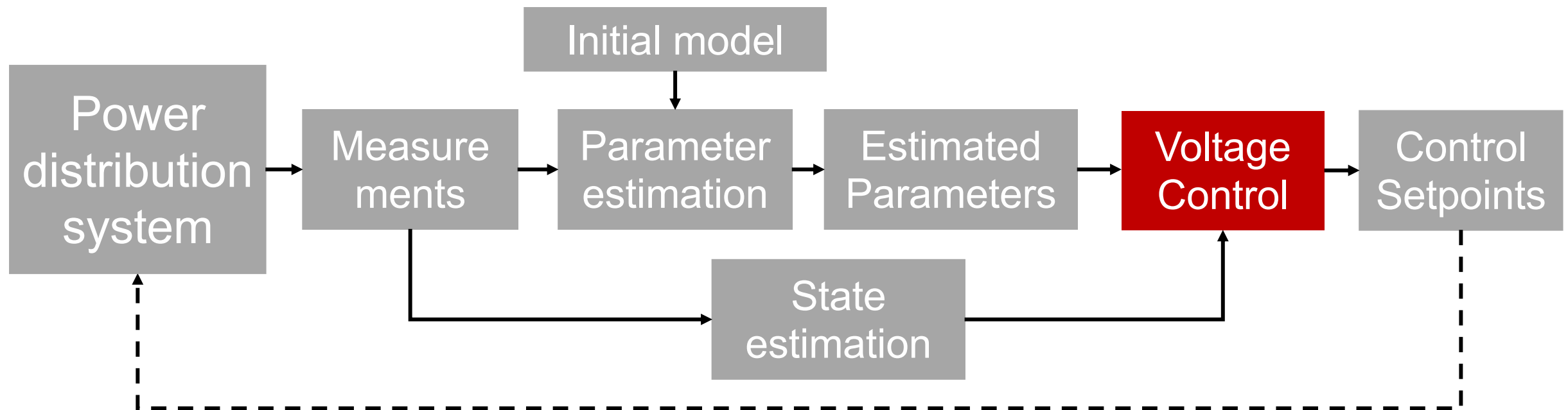
Incorporating Uncertainty into Decision Making



Measurement-based Control on “Invisible” or Partially known Distribution System

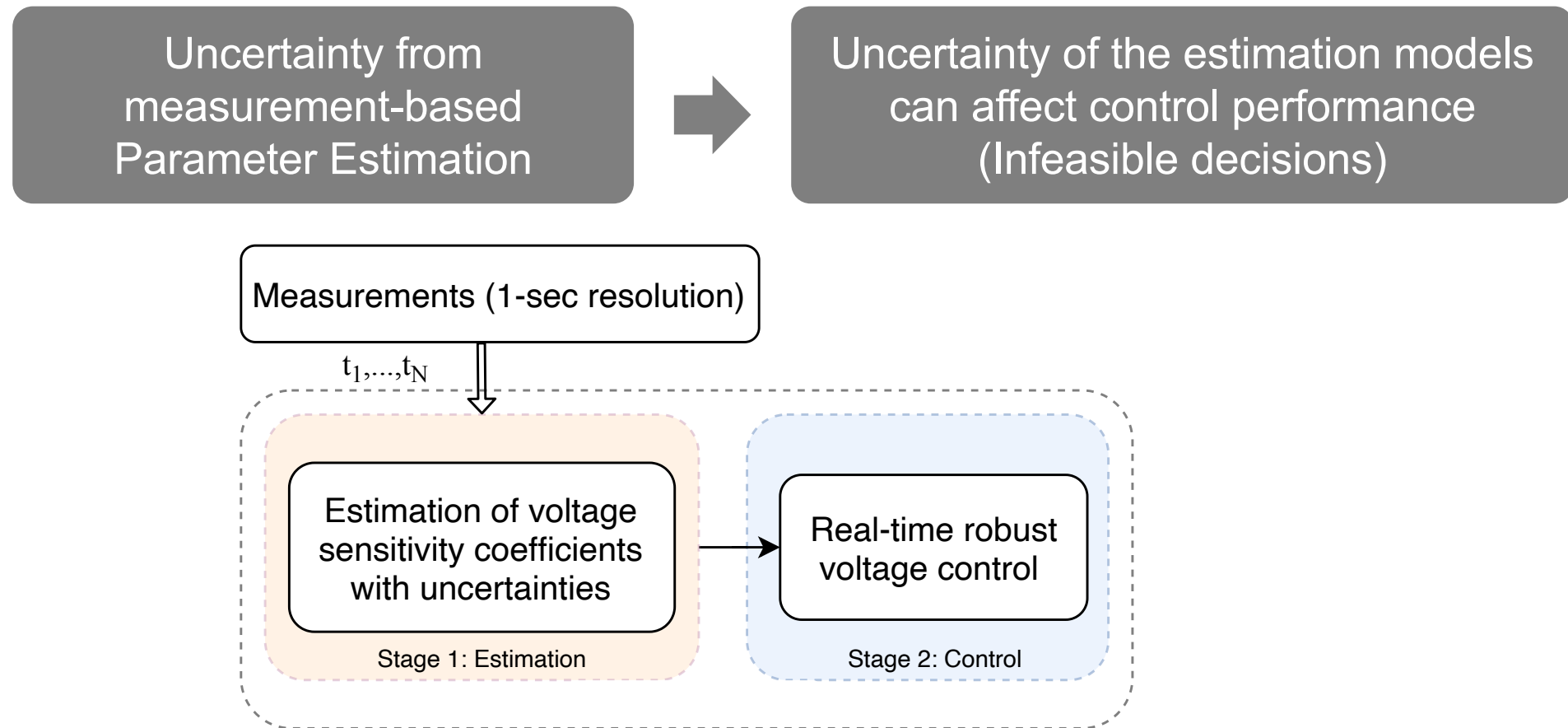
Consider a distribution network whose network parameters are not accurately known.

Objective: to develop a **measurement-based** and **real-time** framework for **controlling distributed energy resources** in distribution systems (e.g., for voltage control). **Experimentally validate** on a real operational network.



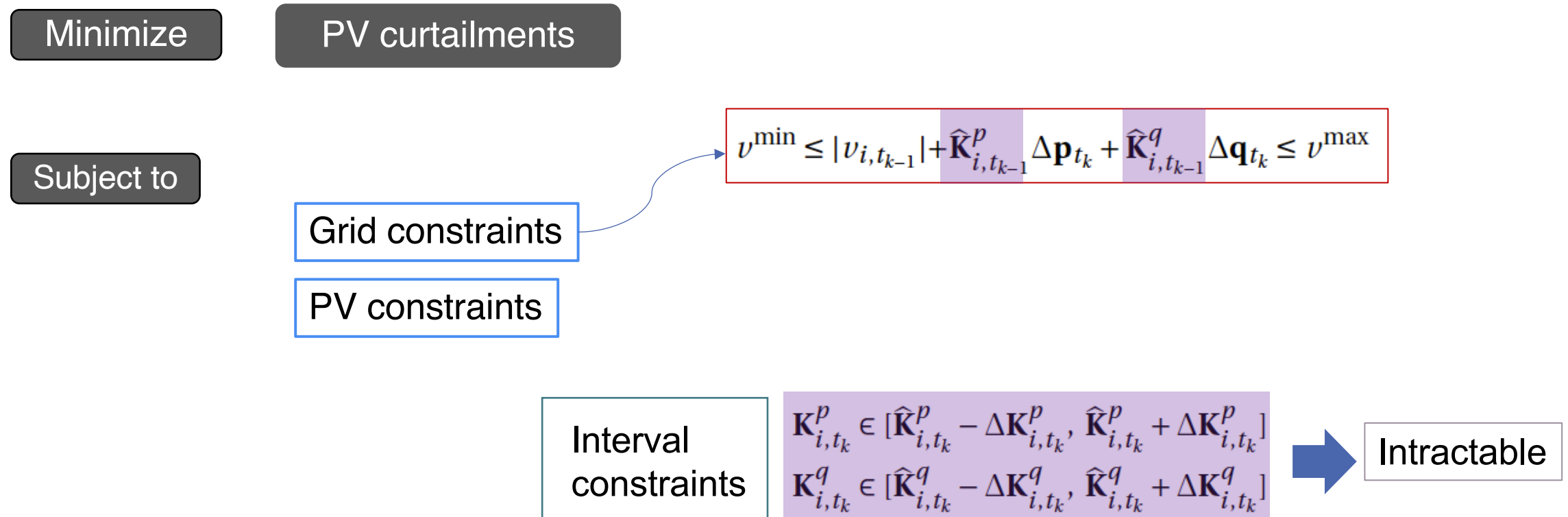
Parameter-uncertainty aware Voltage Control

- This work proposed model-less and measurement-based robust control scheme accounting for the uncertainty of the estimated models.



Voltage Control Example

- Consider a case of distribution system facing overvoltage issues due to overproduction from their PV plants



Gupta, R., Sossan, F., & Paolone, M. (2022). Model-less robust voltage control in active distribution networks using sensitivity coefficients estimated from measurements. *Electric Power Systems Research*, 212, 108547.



Robust Control Formulation

Introduce auxiliary variables $z_i, g_{ij}, y_j^p, y_j^q, j \in \mathcal{N}^b, i \in \mathcal{N}^{pv}$ [Bertsimas et. al 2004], nodal voltage constraint can be re-written as:

$$|v_{i,t_{k-1}}| + \hat{\mathbf{K}}_{i,t_{k-1}}^p \Delta \mathbf{p}_{t_k} + \hat{\mathbf{K}}_{i,t_{k-1}}^q \Delta \mathbf{q}_{t_k} + z_i \xi_i + \sum_{j \in \mathcal{N}_{pv}} g_{ij} \leq v^{\max} \quad \forall i \in \mathcal{N}$$

$$|v_{i,t_{k-1}}| + \hat{\mathbf{K}}_{i,t_{k-1}}^p \Delta \mathbf{p}_{t_k} + \hat{\mathbf{K}}_{i,t_{k-1}}^q \Delta \mathbf{q}_{t_k} - z_i \xi_i - \sum_{j \in \mathcal{N}_{pv}} g_{ij} \geq v^{\min} \quad \forall i \in \mathcal{N}$$

$$-y_j^p \leq \Delta p_{j,t_k}^{pv} \leq y_j^p \quad \forall j \in \mathcal{N}_{pv}$$

$$-y_j^q \leq \Delta q_{j,t_k}^{pv} \leq y_j^q \quad \forall j \in \mathcal{N}_{pv}$$

$$z_i + g_{ij} \geq \Delta K_{ij,t_k}^p y_j^p \quad i \in \mathcal{N}, j \in \mathcal{N}_{pv}$$

$$z_i + g_{ij} \geq \Delta K_{ij,t_k}^q y_j^q \quad i \in \mathcal{N}, j \in \mathcal{N}_{pv}$$

$$y_j^p, y_j^q, z_i, g_{ij} \geq 0 \quad i \in \mathcal{N}, j \in \mathcal{N}_{pv}.$$

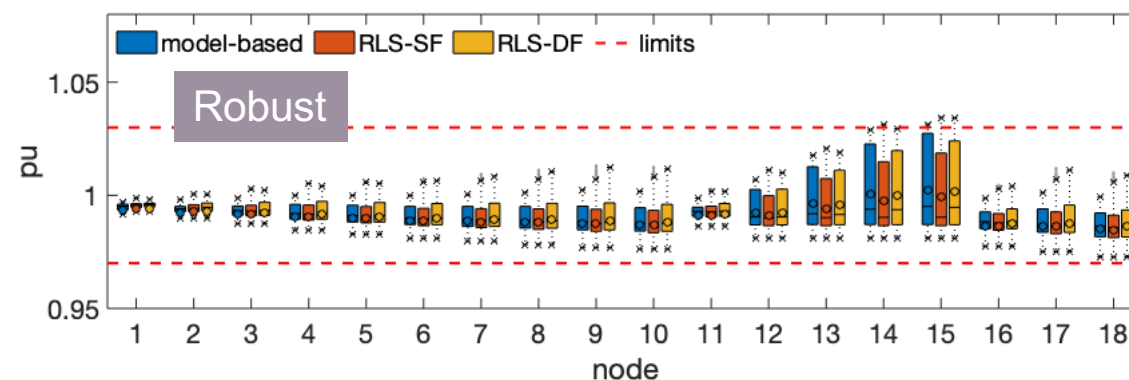
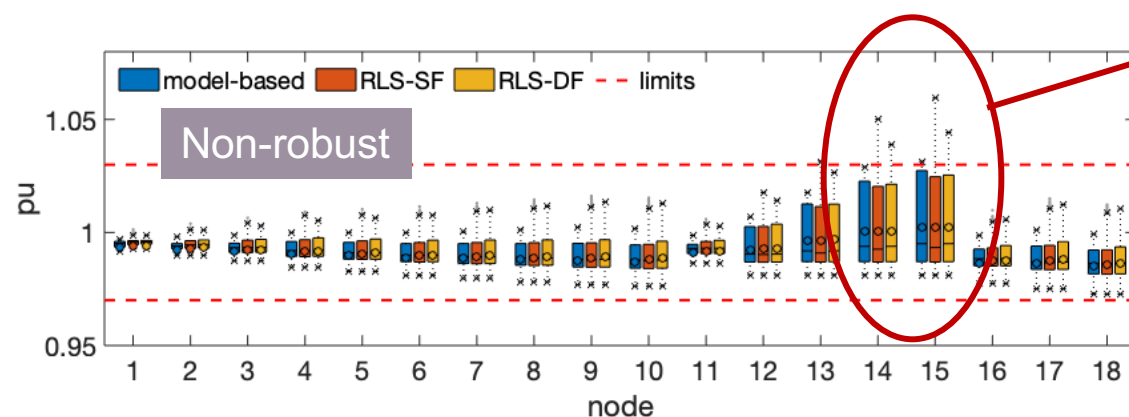
D. Bertsimas and M. Sim, "The price of robustness," Operations research, vol. 52, no. 1, pp. 35–53, 2004.

Gupta, R., Sossan, F., & Paolone, M. (2022). Model-less robust voltage control in active distribution networks using sensitivity coefficients estimated from measurements. *Electric Power Systems Research*, 212, 108547.

- This becomes a simple linear program (LP).

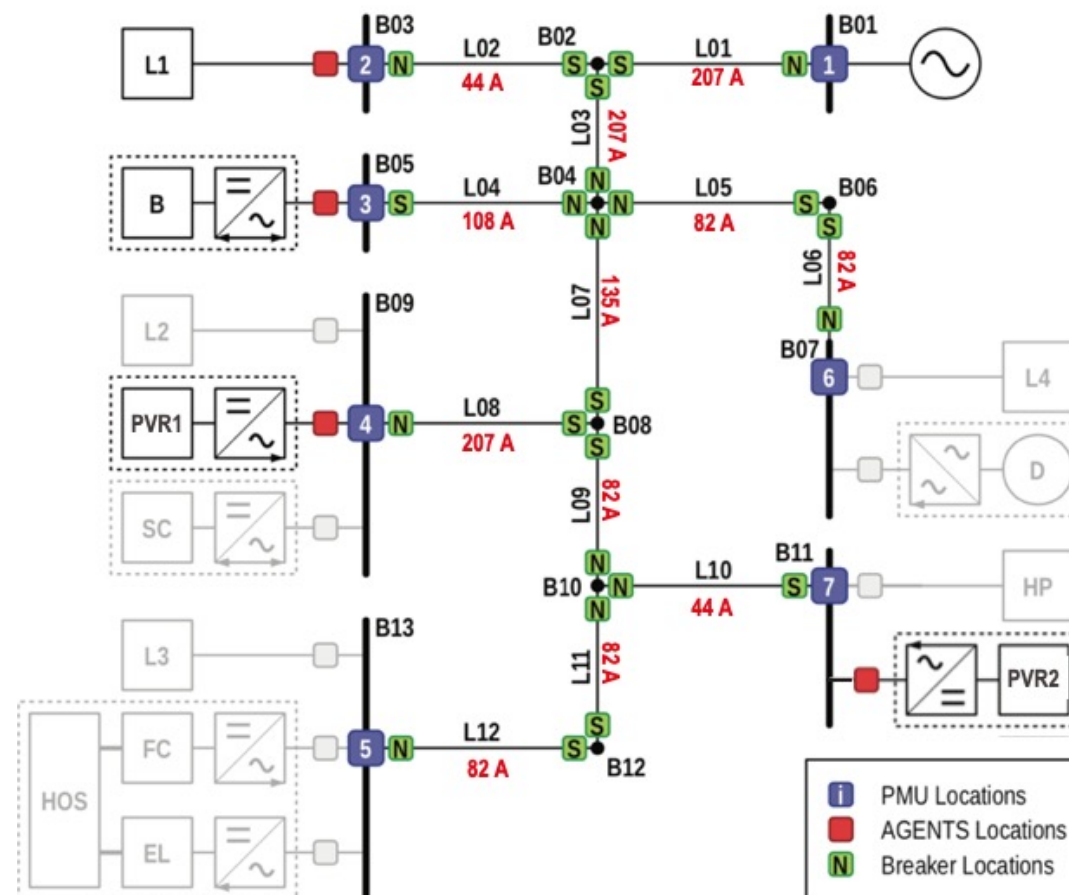


Performance Comparison: Simulation



	Non-robust	Robust
Model-based	1.031	
RLS-DF	1.05	1.034

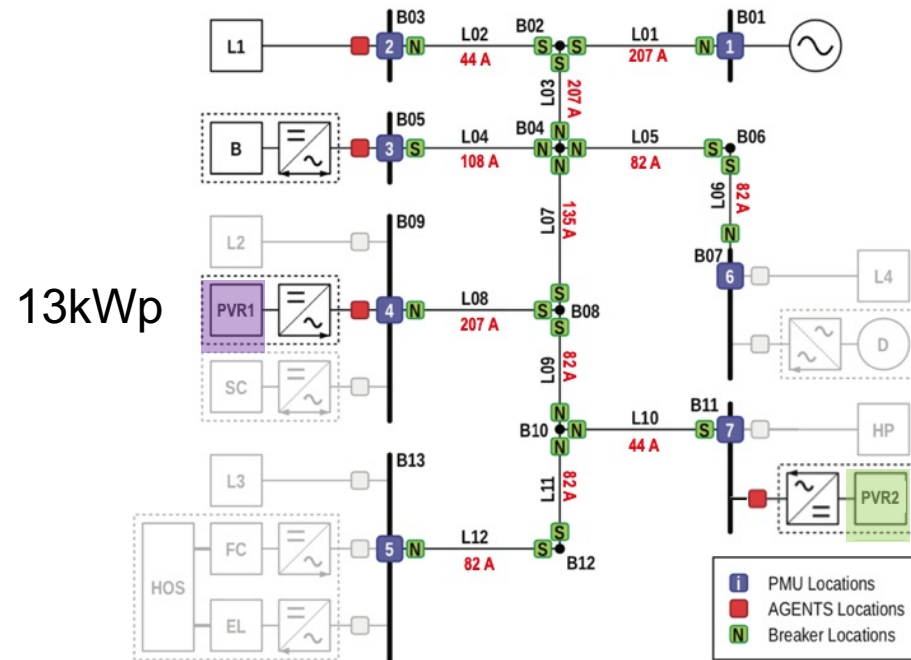
Voltage violations



Gupta, R., Sossan, F., & Paolone, M. (2022). Model-less robust voltage control in active distribution networks using sensitivity coefficients estimated from measurements. *Electric Power Systems Research*, 212, 108547.

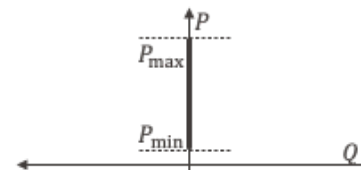
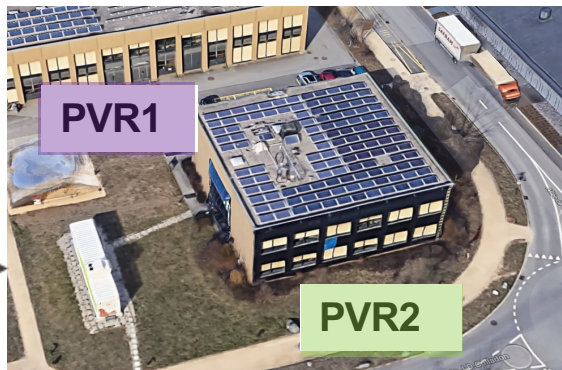
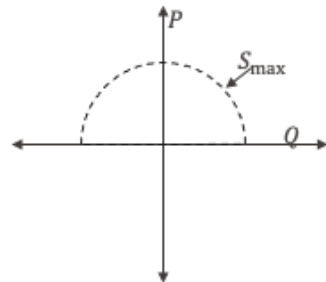


Experimental Setup: Microgrid at EPFL, Switzerland

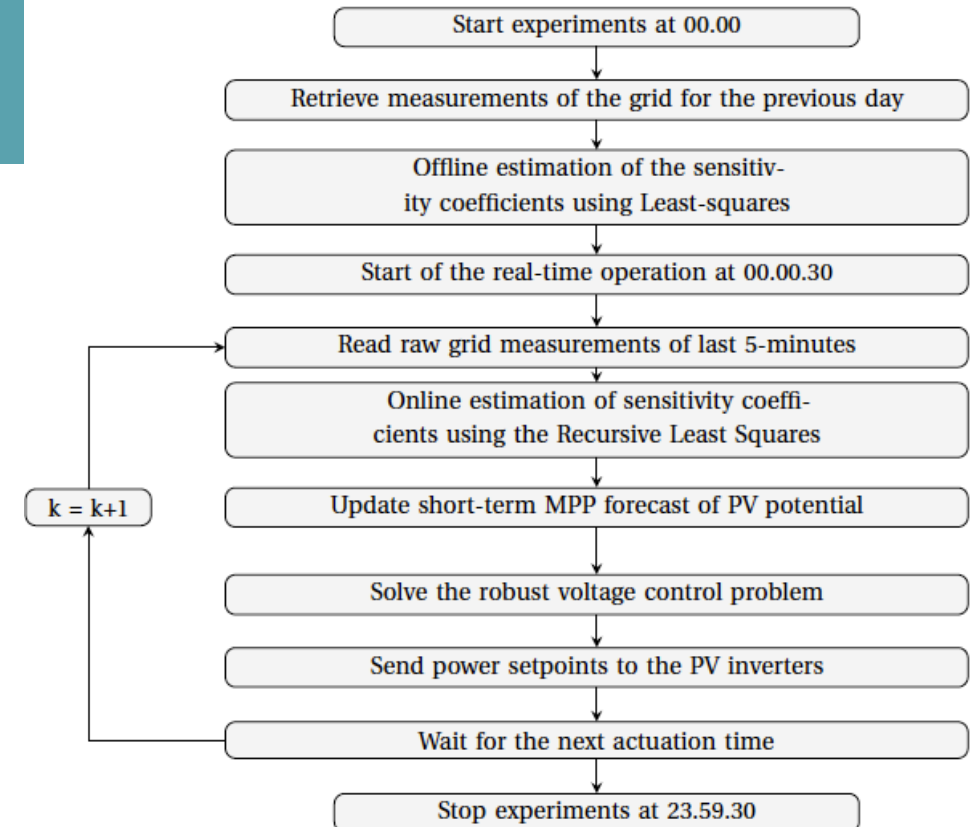


- Actuate the two PV inverters at each 30s

PV1: can operate only in 2 quadrants.



PV2: only active power is controllable.



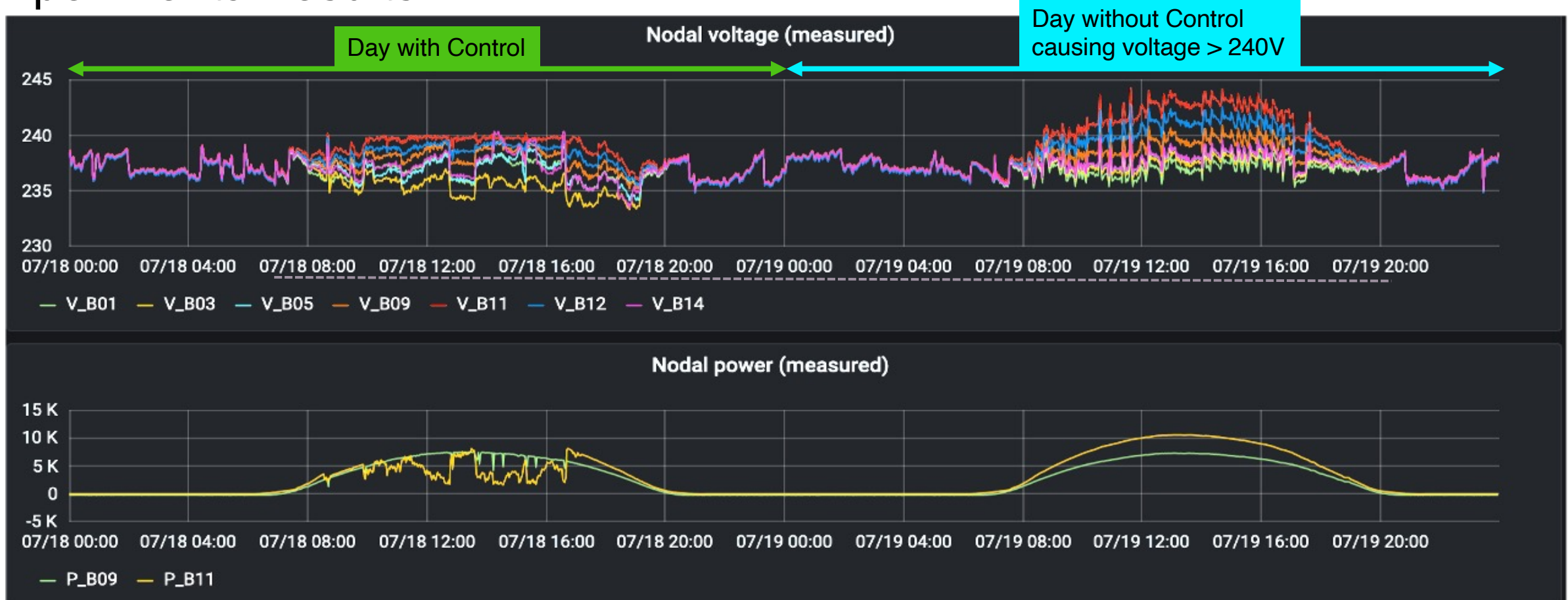
Gupta, R., & Paolone, M. (2023, June). Experimental validation of model-less robust voltage control using measurement-based estimated voltage sensitivity coefficients. In *2023 IEEE Belgrade PowerTech* (pp. 1-8). IEEE.



Experimental Validation: Microgrid at EPFL, Switzerland



Experimental results:



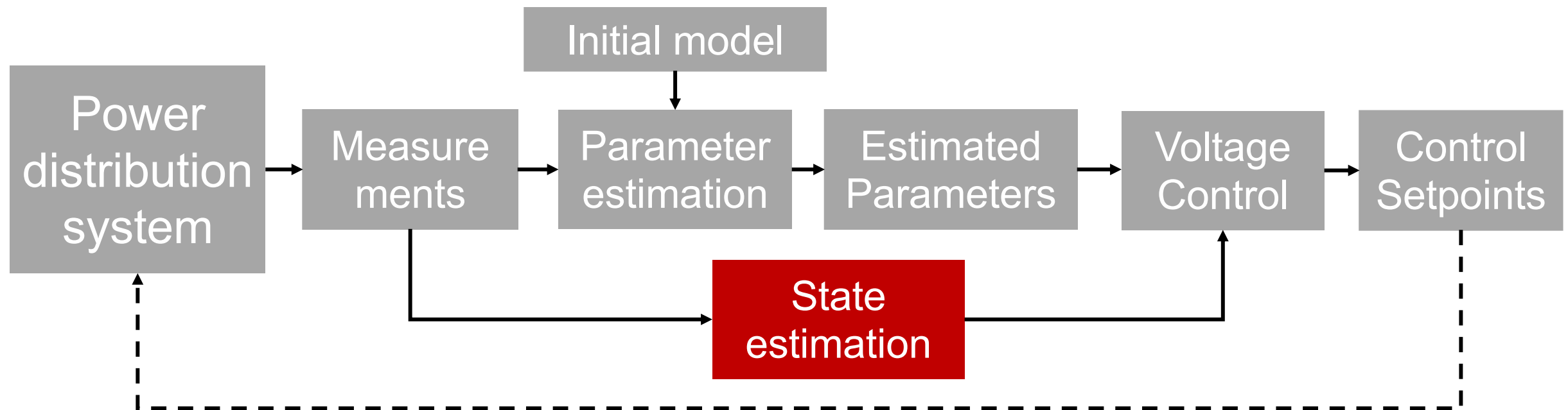
Gupta, R., & Paolone, M. (2023, June). Experimental validation of model-less robust voltage control using measurement-based estimated voltage sensitivity coefficients. In *2023 IEEE Belgrade PowerTech* (pp. 1-8). IEEE.



Measurement-based Control on “Invisible” or Partially known Distribution System

Consider a distribution network whose network parameters are not accurately known.

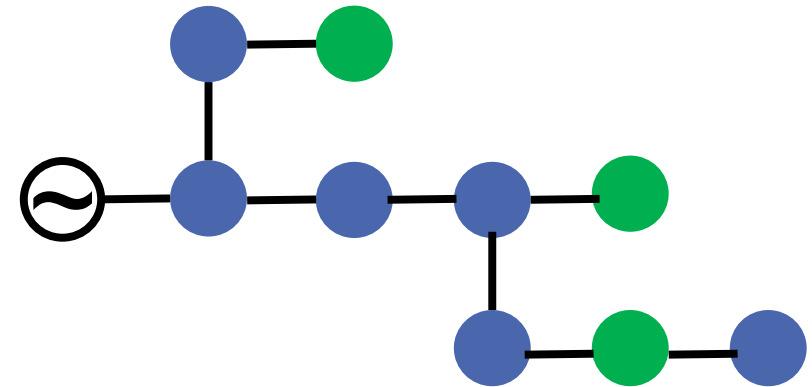
Objective: to develop a **measurement-based** and **real-time** framework for **controlling distributed energy resources** in distribution systems (e.g., for voltage control). **Experimentally validate** on a real operational network.



State Estimation under Low Observability

- **Assumptions –**

- The topology of the network **is known**.
- Smart meters installed at **all nodes with non-zero power injections**.
- **Subset of measurements can be accessed** due to limited communication bandwidth.
- **Dynamically varying** measurement set.
- **Normally distributed** random variables.



Inaccessible smart meters (due to communication limits)

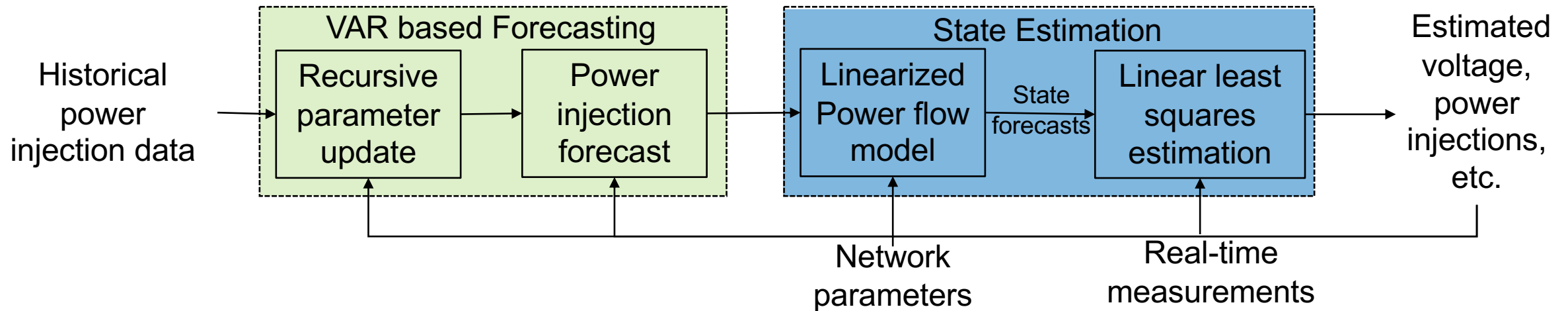


Accessible smart meter measurements

Rangarajan, A., Gupta, R. K., Molzahn, D. K., & Roald, L. (2025). Forecast-aided state estimation in unbalanced distribution networks using smart meter data under limited communication bandwidth. *Sustainable Energy, Grids and Networks*, 101863.



Forecasting-aided State Estimation

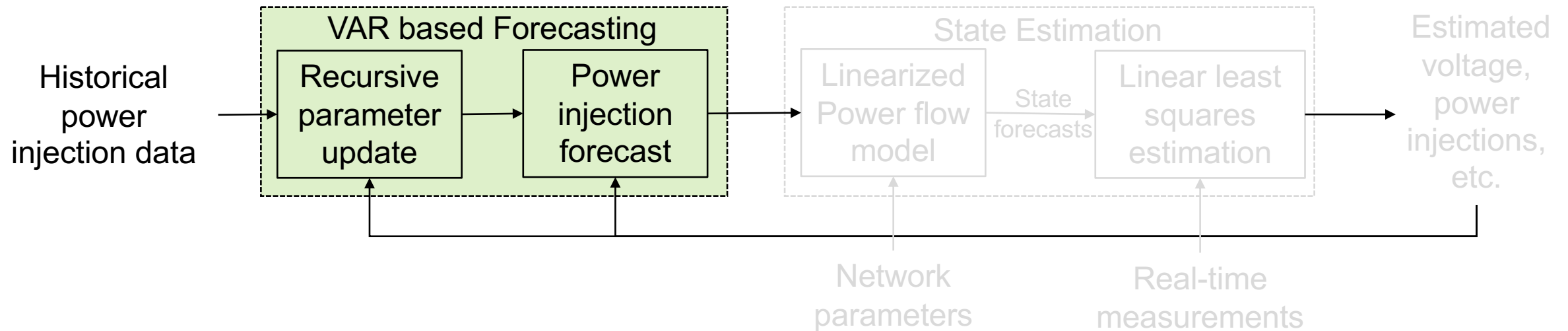


*Key Idea: Perform **state estimation** by utilizing **power injection forecasts from unobserved nodes** to supplement **real-time measurements** from a small set of observed nodes*

Rangarajan, A., Gupta, R. K., Molzahn, D. K., & Roald, L. (2025). Forecast-aided state estimation in unbalanced distribution networks using smart meter data under limited communication bandwidth. *Sustainable Energy, Grids and Networks*, 101863.



Power Injection Forecasting



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Forecast model training: Power Injection

Nodal Power Injections are modelled using the **Vector Auto Regressive (VAR)** model

Recursive Least Squares (RLS)

$$\hat{B}_t = \hat{B}_{t-1} + K_t((\tilde{S}_t)^T - (\tilde{S}_{t-1})^T \hat{B}_{t-1})$$

where K_t is obtained by solving

$$\min ||B - \hat{B}||_F^2$$

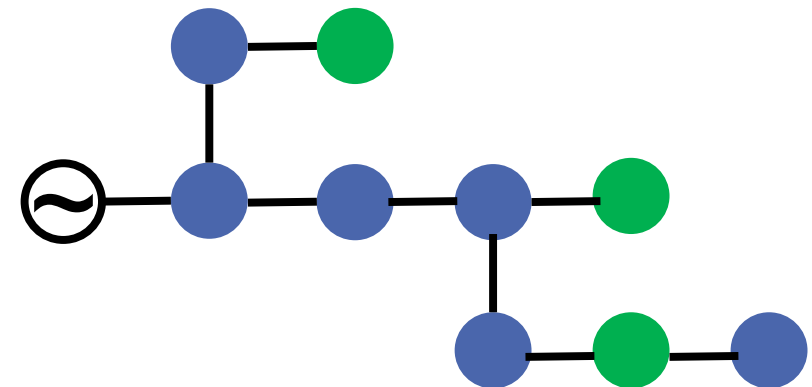
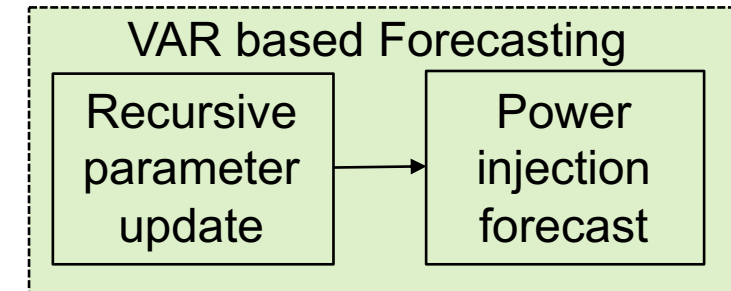
Initial parameter estimates obtained during **an offline training phase**

Observed nodes M_i

$$\tilde{S}_{t,m} = S_{t,m} \quad \forall m \in M_i$$

Unobserved nodes: persistence model
(quasi-steady state power injections)

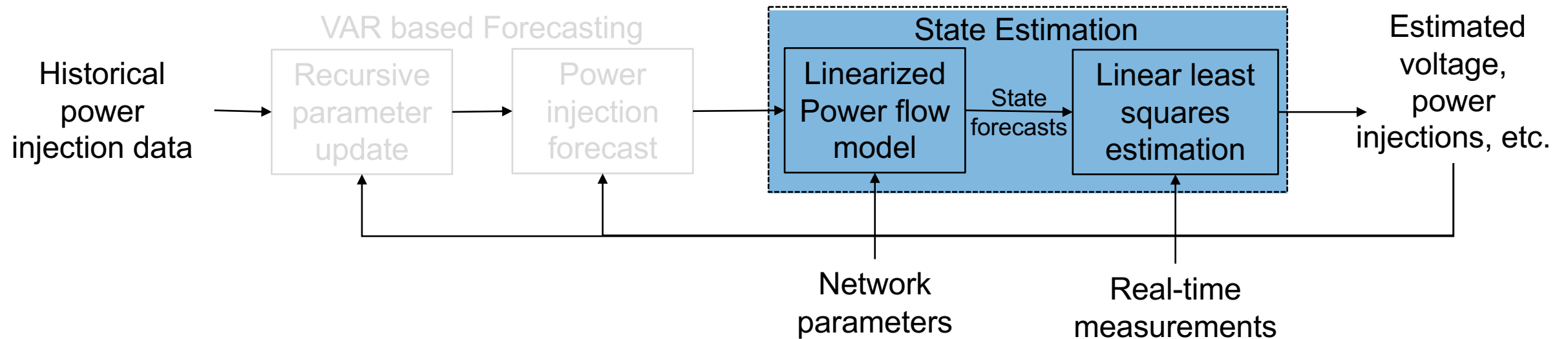
$$\tilde{S}_{t,m} = \tilde{S}_{t-1,m} \quad \forall m \notin M_i$$



- Inaccessible smart meters (due to communication limits)

● Accessible smart meter measurements

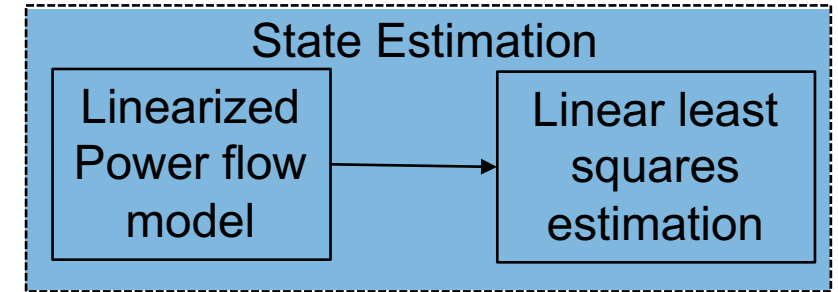
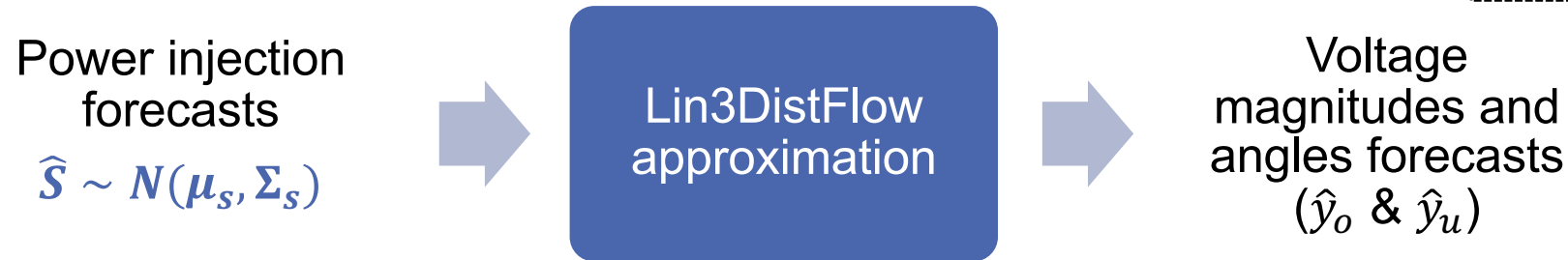
State Estimation Model



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Forecasting system states & measurements



- Measurement Forecasts

$$\hat{y}_o^m = \hat{y}_o + \eta$$
$$\hat{y}_o^m \sim N(\mu_o, \Sigma_o + \Omega_o)$$

- Computed means (μ_o & μ_u) and covariances (Σ_o , Σ_u , & Σ_{uo}) are **used as priors** to **perform state estimation**

M. D. Sankur, R. Dobbe, A. v. Meier, E. M. Stewart, and D. B. Arnold, "Optimal voltage phasor regulation for switching actions in unbalanced distribution systems," in IEEE Power & Energy Society General Meeting (PESGM), 2020.



Linear least squares estimation

- *Goal: Minimize the expectation of the squared error*

$$\min \mathbb{E} \left(\|\tilde{y}_u - y_u\|^2 \right)$$

where $\tilde{y}_u = g(\hat{y}_o^m)$ is the state estimate obtained using measurements

- Gaussian Random variables $\Rightarrow$ Linear State Estimator $\tilde{y}_u = G\hat{y}_o^m$

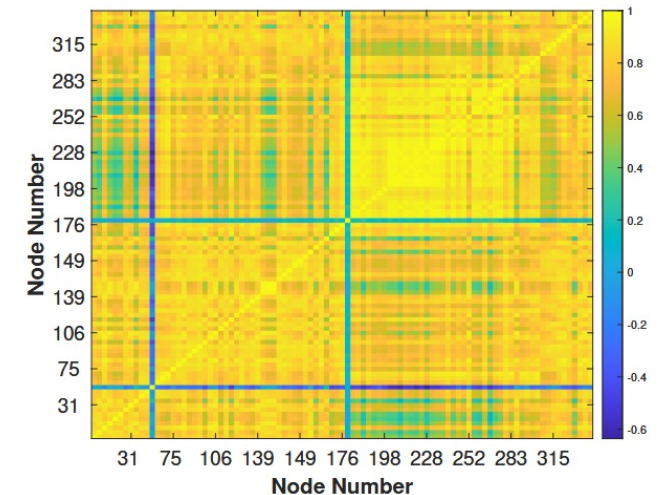
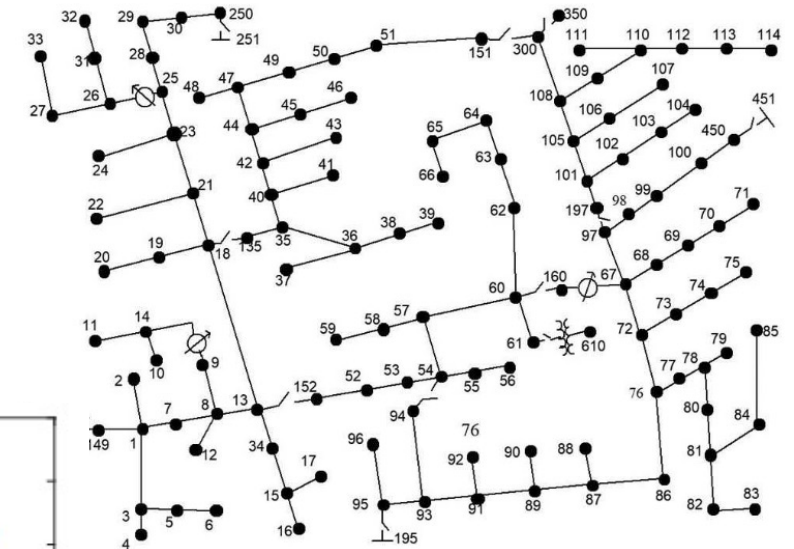
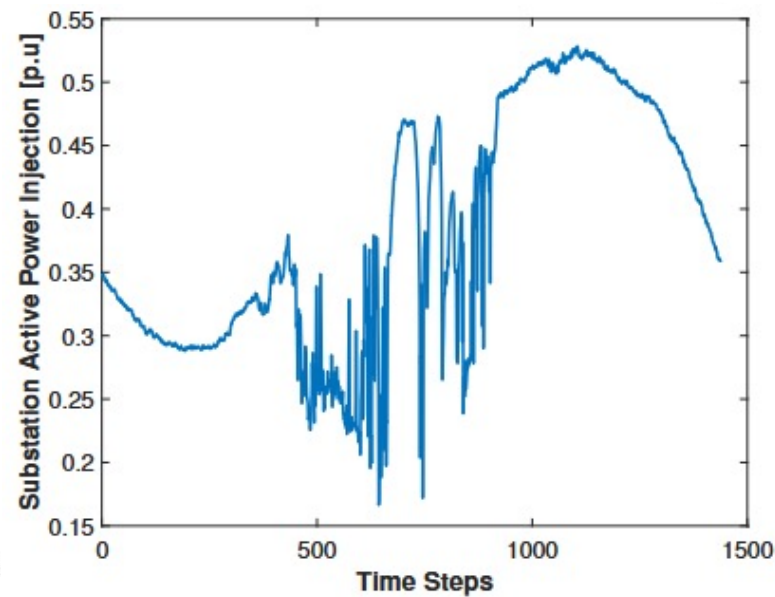
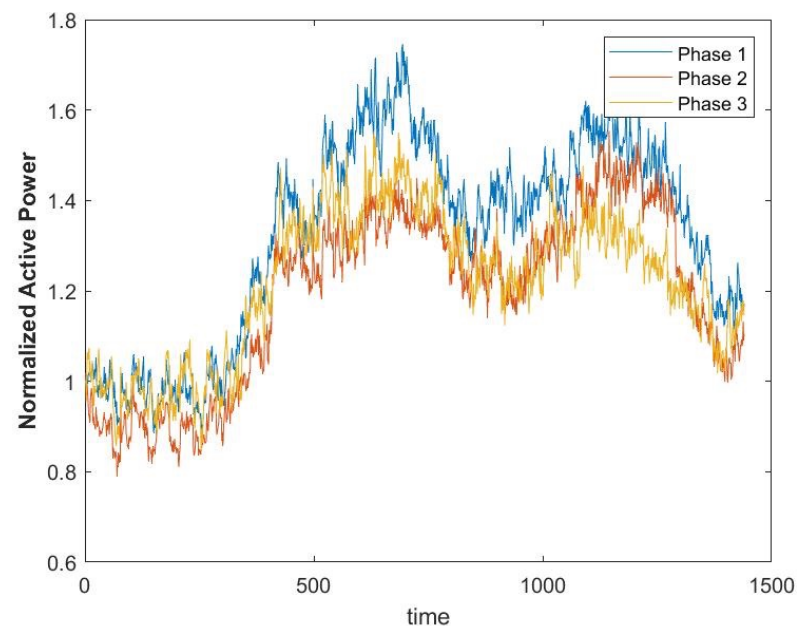
$$\tilde{y}_u^* = \mathbb{E}(\hat{y}_u | \hat{y}_o^m) = \mu_u + \Sigma_{uo}(\Sigma_o + \Omega_o)^{-1}(y_o^m - \mu_o)$$

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Case Study – Base case

- IEEE 123 bus feeder
 - Load profiles **for an entire day** from a 6-feeder distribution network in Portland, OR [3]
 - Load profiles represent a **small geographic region** with **low renewable generation**

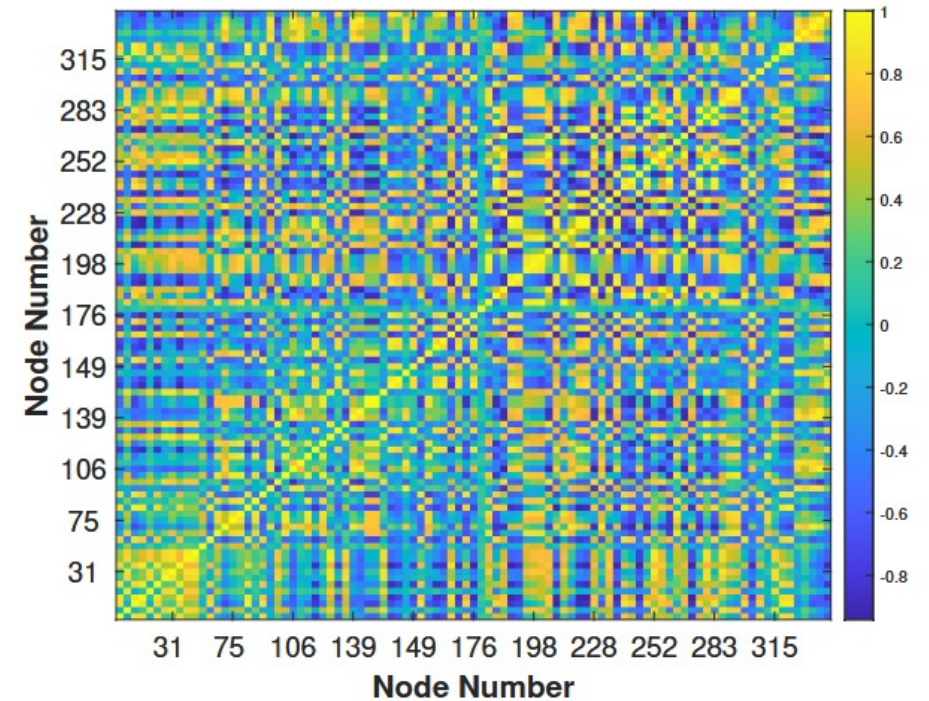


Y. Gu, "Meteorological and customer composition factors in power load forecasting," Masters Thesis, University of Wisconsin–Madison, 2023.



Case Study – Low Correlation

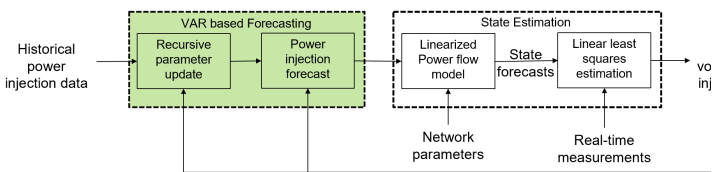
- Small geographical region $\Rightarrow$ Highly correlated power consumption
- Ensure that the method performs well in **scenarios with low correlation**
- Power injection data generated **by shuffling** each load profile **randomly in time**



Correlation in Active power injection

Results – Power injection forecasting

Recursive least squares **accurate** even with **10% measured nodes** and with access to **limited historical data**



Method	Mean Absolute Error [p.u.]	Root Mean Square Error [p.u.]
Yule-Walker	1.40×10^{-3}	2.14×10^{-3}
Recursive Least Squares (100% measurements)	6.74×10^{-4}	4.45×10^{-4}
Recursive Least Squares (10% measurements)	7.01×10^{-4}	4.47×10^{-4}

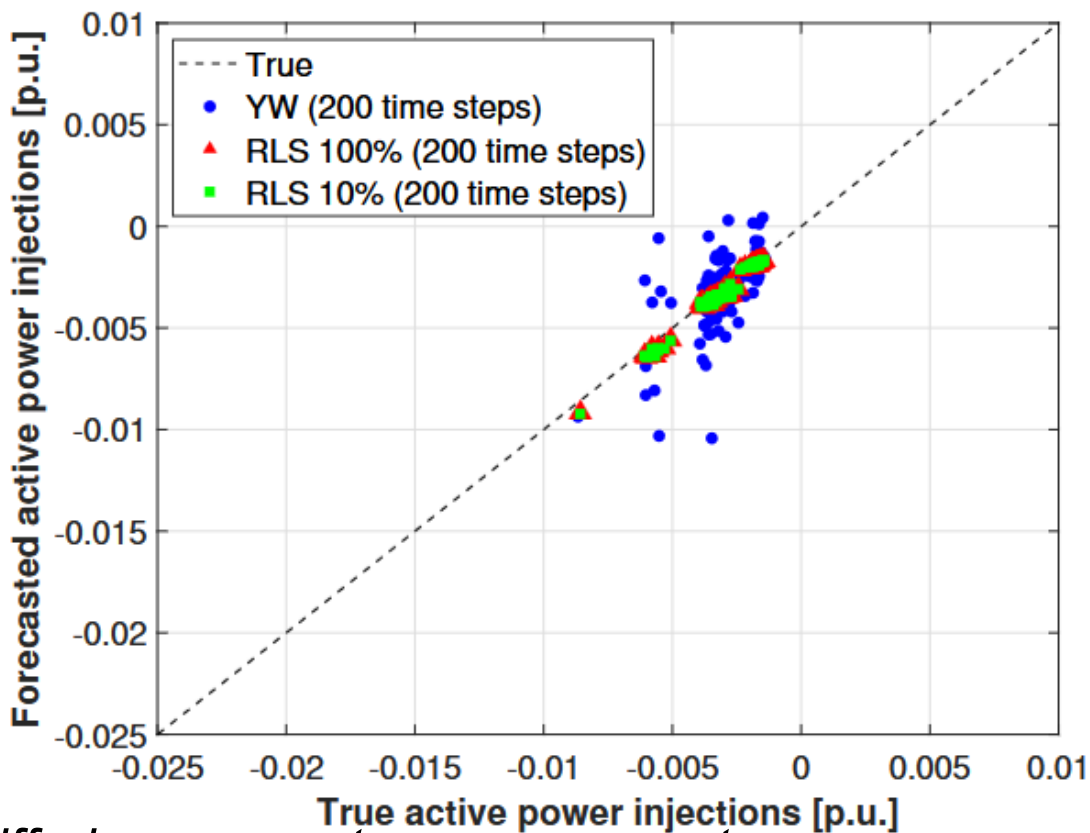
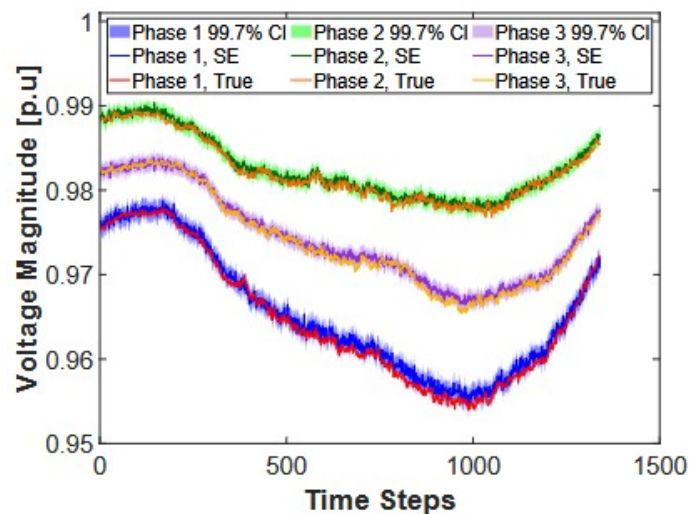
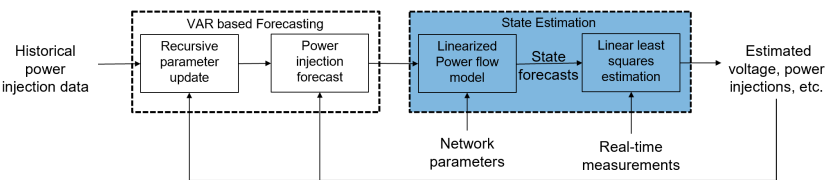


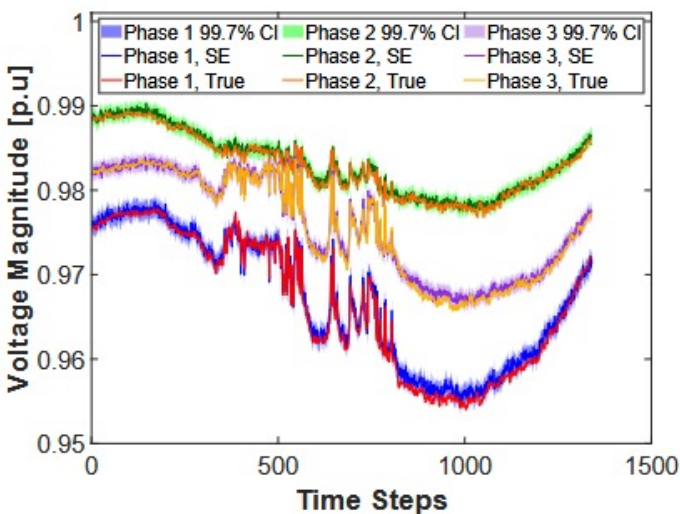
Table 1: Forecasting accuracy with differing access to measurements
(Yule-Walker method serves as a benchmark)



Results – State estimation



(d) BC: 10% measurements



(e) PV: 10% measurements

Scenarios	Mean Absolute Error [p.u.]	Root Mean Square Error [p.u.]
Base case (BC)	4.20×10^{-4}	5.50×10^{-4}
50% Renewables case (PV)	3.88×10^{-4}	5.14×10^{-4}

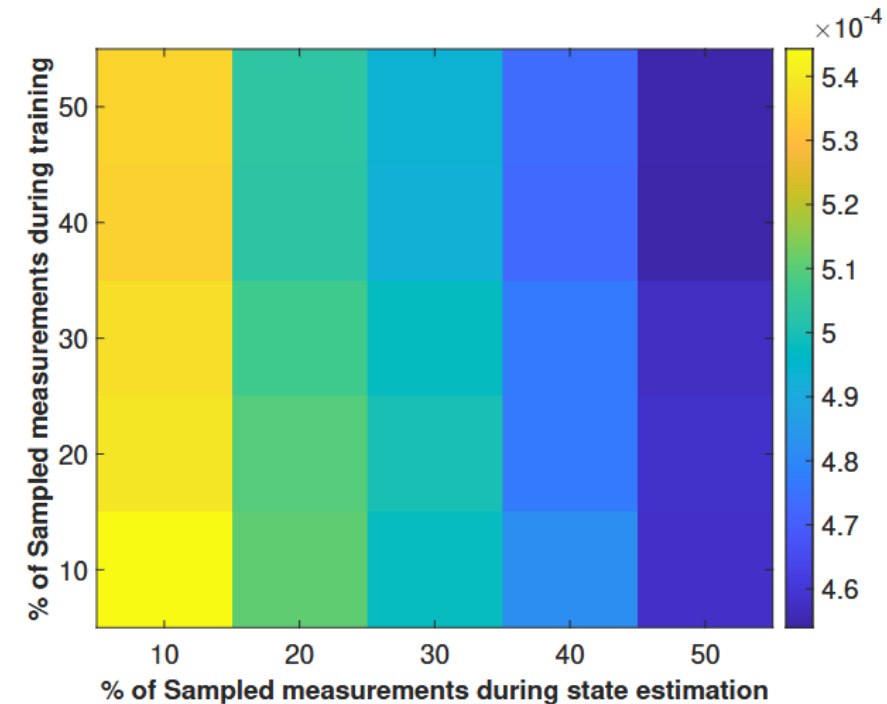
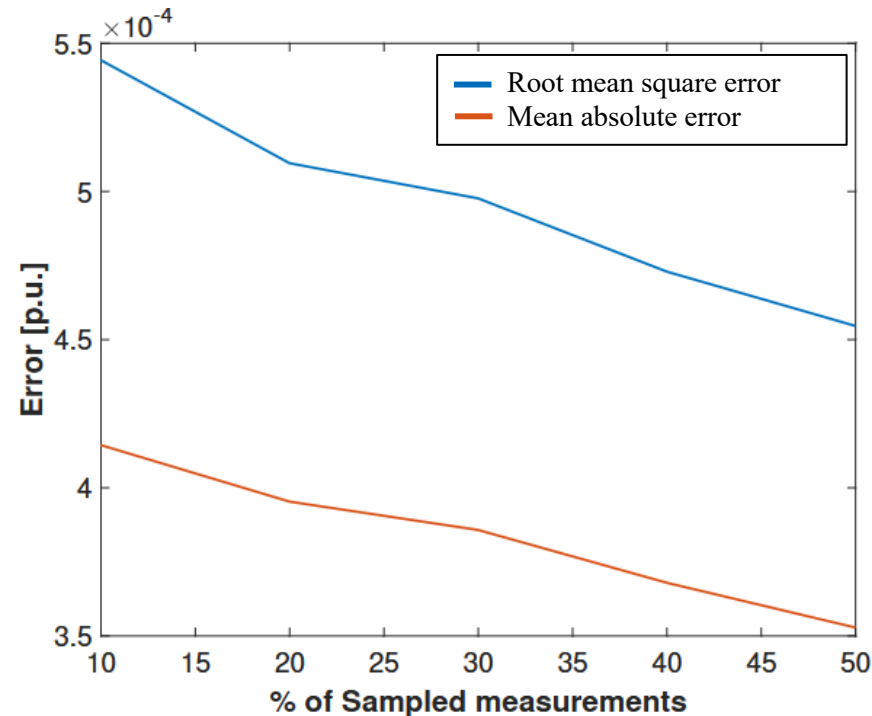
Table: Voltage estimation errors in different scenarios

The proposed scheme accurately estimates system states with **measurements from only 10% of nodes**



Results – Impact of number of measurements

Performance **improves** with **more measurements**.



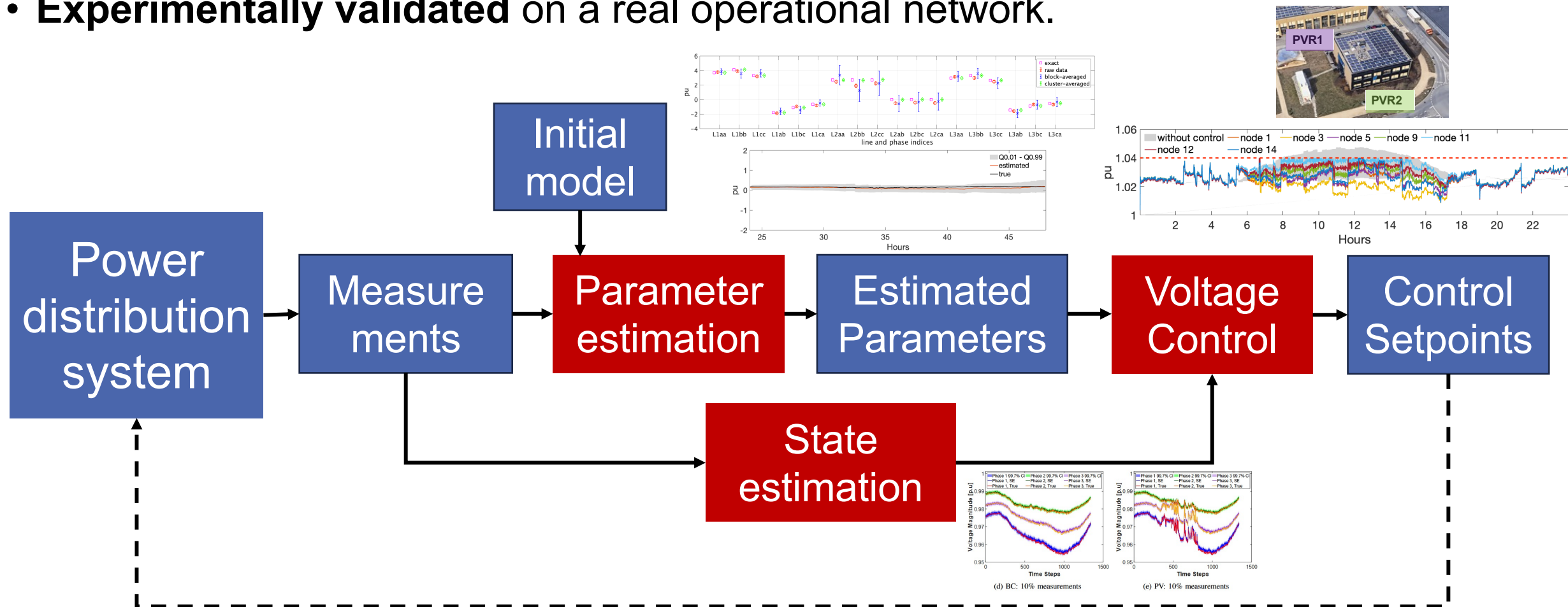
Performance is **more sensitive** to the measurements accessed during the **real-time state estimation stage**.

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Summary

- We developed a **measurement-based** and **real-time** framework for **controlling distributed energy resources** in distribution systems (e.g. for voltage control).
- **Experimentally validated** on a real operational network.



Thank You for Attending!

Review paper on Power System Estimation Techniques:

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Assistant Professor,
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Group: <https://rahul-kgupta.github.io>

Learning Power Flow Models and Constraints From Time-Synchronized Measurements: A Review

This article presents a comprehensive review of measurement-based estimation methods for power flow models using time-synchronized measurements.

By RAHUL K. GUPTA[✉], Member IEEE, PAOLO ATTILIO PEGORARO[✉], Senior Member IEEE, OGNJEN STANOJEV, Student Member IEEE, ALI ABUR[✉], Life Fellow IEEE, CARLO MUSCAS[✉], Senior Member IEEE, GABRIELA HUG[✉], AND MARIO PAOLONE[✉] Fellow IEEE



Time for Q&A!

